Supplemental Appendix: Not Intended for Publication

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A Sufficient Conditions for a Stable Interior Equilibria

In this appendix, we derive sufficient conditions for a stable, interior spatial equilibrium. To do so, we define the equation of motion for workers as the gap between the probability that a worker of type s chooses to locate in city i and the actual share of type s workers in city i. Specifically, denote

$$\begin{aligned} \dot{n}_{s1} &= Prob(U_{s1} > U_{s2}) - \frac{n_{s1}}{n}, \\ &= \frac{V_{s1}^{\theta}}{V_{s1}^{\theta} + V_{s2}^{\theta}} - \frac{n_{s1}}{n}, \end{aligned} \tag{A.1}$$

where $\dot{n}_{s1}$ denotes the time derivative type s workers in city 1. For reference, we rewrite the functions for V_{si} below.

$$\begin{aligned} V_{hi} &= \kappa A_i \frac{n_{hi}^{\rho+\eta-\frac{1}{\sigma}}}{(n_{hi}+n_{li})^{\chi}} \left(n_{hi}^{1+\eta-\frac{1}{\sigma}} + n_{li}^{\frac{\sigma-1}{\sigma}} \right)^{\frac{1-\mu(1-\beta)\sigma}{\sigma-1}} L_i^{\mu(1-\beta)}, \\ V_{li} &= \kappa A_i \frac{n_{li}^{-\frac{1}{\sigma}}}{(n_{hi}+n_{li})^{\chi}} \left(n_{hi}^{1+\eta-\frac{1}{\sigma}} + n_{li}^{\frac{\sigma-1}{\sigma}} \right)^{\frac{1-\mu(1-\beta)\sigma}{\sigma-1}} L_i^{\mu(1-\beta)}. \end{aligned} \tag{A.2}$$

An interior equilibrium is a value of $n_{si} \in (0, n)$ such that $\dot{n}_{s1} = 0$. The interior equilibrium is stable if the eigenvalues associated with the Jacobian matrix, defined by

$$J \equiv \begin{bmatrix} \frac{\partial \dot{n}_{h1}}{\partial n_{h1}} & \frac{\partial \dot{n}_{h1}}{\partial n_{l1}} \\ \frac{\partial \dot{n}_{l1}}{\partial n_{h1}} & \frac{\partial \dot{n}_{l1}}{\partial n_{l1}}, \end{bmatrix},$$

are both negative. We focus on the symmetric equilibrium where both regions are ex-ante identical such that $n_{si} = n/2$. Sufficient conditions for this to hold are that both terms of the trace of J be negative and the determinant be positive when evaluated at the equilibrium population distribution. This requires that

$$\left(\frac{\partial \dot{n}_{h1}}{\partial n_{h1}} + \frac{\partial \dot{n}_{l1}}{\partial n_{l1}}\right)|_{[n_{si}=n/2]} = (A - B + C) + (D - B + E) < 0, \tag{A.3}$$

$$(\frac{\partial \dot{n}_{h1}}{\partial n_{h1}}\frac{\partial \dot{n}_{l1}}{\partial n_{l1}} - \frac{\partial \dot{n}_{h1}}{\partial n_{l1}}\frac{\partial \dot{n}_{l1}}{\partial n_{h1}})|_{[n_{si}=n/2]} = (A - B + C)(D - B + E) - (-B + C)(-B + E),$$

$$= AD + A(-B + E) + D(-B + C) > 0 \tag{A.4}$$

where

$$A \equiv (\rho + \eta) - (\frac{1}{\sigma} + \frac{1}{\theta}) \gtreqless 0, \quad B \equiv \frac{\chi}{2} > 0, \quad C \equiv \left(\frac{1 - \mu(1-\beta)\sigma}{\sigma - 1}\right)\frac{(\eta + \frac{\sigma-1}{\sigma})\left(\frac{n}{2}\right)^{\eta}}{\left(\frac{n}{2}\right)^{\eta} + 1} > 0$$

$$D \equiv -(\frac{1}{\sigma} + \frac{1}{\theta}) < 0, \quad E \equiv \left(\frac{1 - \mu(1-\beta)\sigma}{\sigma - 1}\right)\frac{\frac{\sigma-1}{\sigma}}{\left(\frac{n}{2}\right)^{\eta} + 1} > 0. \tag{A.5}$$

We now show that these conditions will be met if χ, which governs the strength of urban costs, is sufficiently large and that the agglomeration forces, $\rho + \eta$ are not too strong relative to the dispersion forces, $\frac{1}{\sigma} + \frac{1}{\theta}$. Inserting the terms in (A.5) into (A.3) and rearranging yields

$$\chi > \left((\rho + \eta) - 2(\frac{1}{\sigma} + \frac{1}{\theta})\right) + \left(\frac{1 - \mu(1-\beta)\sigma}{\sigma - 1}\right)\frac{(\eta + \frac{\sigma-1}{\sigma})\left(\frac{n}{2}\right)^{\eta} + \frac{\sigma-1}{\sigma}}{\left(\frac{n}{2}\right)^{\eta} + 1}. \tag{A.6}$$

Inserting (A.5) into (A.4) yields the condition

$$\frac{\chi}{2}\left(2(\frac{1}{\sigma} + \frac{1}{\theta}) - (\rho + \eta)\right) > (\frac{1}{\sigma} + \frac{1}{\theta})(\rho + \eta - \frac{1}{\sigma} - \frac{1}{\theta})$$

$$+\left(\frac{1 - \mu(1-\beta)\sigma}{\sigma - 1}\right)\frac{((\frac{1}{\sigma} + \frac{1}{\theta}) - (\rho + \eta))\frac{\sigma-1}{\sigma} + (\frac{1}{\sigma} + \frac{1}{\theta})(\eta + \frac{\sigma-1}{\sigma}\left(\frac{n}{2}\right)^{\eta}}{\left(\frac{n}{2}\right)^{\eta} + 1}.$$

Recalling the definition $\psi \equiv ((\frac{1}{\sigma} + \frac{1}{\theta}) - (\eta + \rho))/(\frac{1}{\sigma} + \frac{1}{\theta}) = (\psi_d - \psi_a)/\psi_d$, we can rewrite the above inequality as

$$\chi(1 + \psi) > 2(\psi_a - \psi_d) + 2\left(\frac{1 - \mu(1-\beta)\sigma}{\sigma - 1}\right)\frac{\psi\frac{\sigma-1}{\sigma} + (\eta + \frac{\sigma-1}{\sigma})\left(\frac{n}{2}\right)^{\eta}}{\left(\frac{n}{2}\right)^{\eta} + 1}. \tag{A.7}$$

The condition in (A.7) provides a lower bound on χ if $1 + \psi > 0 \implies 2(1/\sigma + 1/\theta) - (\eta + \rho) > 0$. Thus, the condition will hold, provided that the sum of the agglomeration parameters does not exceed twice the value of the sum of the inverse of the dispersion parameters. This condition together with the bounds laid out in (A.6) and (A.7) provides sufficient conditions for a stable interior equilibrium, and we assume that this holds throughout the analysis.

A.1 Existence and Uniqueness

In this section, we consider the general existence and uniqueness of the equilibrium. To begin we rewrite the final equilibrium condition which yields the population distribution for skilled workers.

$$\frac{n_{h1}}{n_{h2}} = \left(\frac{V_{h1}}{V_{h2}}\right)^{\theta} \implies$$

$$\left(\frac{n_{h1}}{n_{h2}}\right)^{\psi_d-\psi_a} = \left(\frac{A_1}{A_2}\right)^{1-\mu(1-\beta)} \left(\frac{n_{h2}+n_{l2}(n_{h1})}{n_{h1}+n_{l1}(n_{h1})}\right)^{\chi} \left(\frac{n_{h1}^{1+\eta-\frac{1}{\sigma}} + n_{l1}(n_{h1})^{\frac{\sigma-1}{\sigma}}}{n_{h2}^{1+\eta-\frac{1}{\sigma}} + n_{l2}(n_{h1})^{\frac{\sigma-1}{\sigma}}}\right)^{\frac{1-\mu(1-\beta)\sigma}{\sigma-1}} \left(\frac{L_1}{L_2}\right)^{\mu(1-\beta)}. \quad \text{(A.8)}$$

Furthermore, recall that

$$\frac{\partial n_{l1}}{\partial n_{h1}} \gtreqless 0 \iff \psi \gtreqless 0. \quad \text{(A.9)}$$

The slope of the functions with respect to n_{h1} on each side of (A.8) depend on the sign of $\psi_d - \psi_a$. Initially, suppose that $\psi_d - \psi_a > 0$ such the dispersion forces dominate. It follows that the LHS of Eq. (A.8) is monotonically increasing in n_{h1} with bounds between 0 and ∞. The RHS will be monotonically decreasing, provided that χ is sufficiently large such that the congestion costs dominate the effect of an increase total city income from a rising population. Furthermore, provided χ is sufficiently strong the RHS will approach ∞ as n_{h1} approaches 0 and 0 as n_{h1} approaches n. This implies that there exists an $n_{hi} \in (0, n)$ that yields a unique point of intersection of the two lines.

In the latter case, the results largely mirror that of Allen and Arkolakis (2014) who show that when congestion forces are sufficiently strong, there is unique and stable interior equilibrium in an economic geography model with a homogenous labor force. However, as pointed out by Farrokhi and Jinkins (2019), their proof does not naturally extend to a model with multiple types of workers. In the case where $\psi_d - \psi_a < 0$, the results regarding existence and uniqueness are less clear. In this scenario, the LHS of (A.8) is decreasing. However, given our assumption that χ is large and that $1+\psi > 0$ from the previous section, the RHS of (A.8) will be decreasing as well. We have undertaken a numerical analysis of (A.8) and find that an interior equilibrium may not exist if cities are sufficiently asymmetric in A_i or L_i. However, even in this scenario, if χ is further increased in response a stable, interior equilibrium can be attained. Furthermore, the parameters ρ and η must not be too high. Thus, our numerical analysis suggests that the basic properties that ensure the existence and stability of the interior equilibrium in the symmetric case continue to hold, namely urban congestion costs must be high and agglomeration forces must not be too strong, but they must adjusted to account for larger asymmetries in first nature differences. Provided these conditions were met we did find evidence of additional asymmetric interior equilibria, but they were found to be unstable. We leave for future research a fuller study of the properties of additional equilibria. In the numerical analysis undertaken in the paper agglomeration economies are sufficiently weak such that the reported results in Tables 3 and 4 are from stable equilibria. We have verified this by checking that eigenvalues associated with the Jacobian matrix from (13) with respect to n_{hi} and n_{li} are negative or have negative real parts.

B Comparative Statics for Result 1

The equilibrium is now defined by

$$\frac{n_{h1}}{n_{h2}} = \left(\frac{V_{h1}}{V_{h2}}\right)^{\theta} \implies$$

$$\left(\frac{n_{h1}}{n_{h2}}\right)^{\psi_d-\psi_a} = \left(\frac{A_1}{A_2}\right)^{1-\mu(1-\beta)\sigma} \left(\frac{n_{h2}+n_{l2}(n_{h1})}{n_{h1}+n_{l1}(n_{h1})}\right)^{\chi} \left(\frac{n_{h1}^{1+\eta-\frac{1}{\sigma}}+n_{l1}(n_{h1})^{\frac{\sigma-1}{\sigma}}}{(n_{h2}^{1+\eta-\frac{1}{\sigma}}+n_{l2}(n_{h1})^{\frac{\sigma-1}{\sigma}}}\right)^{\frac{1-\mu(1-\beta)}{\sigma-1}} \left(\frac{L_1}{L_2}\right)^{\mu(1-\beta)}. \tag{B.1}$$

Totally differentiating (B.1) with respect to n_{h1} and L_1 and using the fact that $n_{l1}'(n_{h1})|_{n_{h1}=n/2}=$ ψ yields

$$\frac{dn_{h1}}{d\bar{L}_1}|_{n_{h1}=n_{l1}=\frac{n}{2}} = \frac{\mu(1-\beta)n}{2L}\left(\frac{1}{\chi(1+\psi)+2(\psi_d-\psi_a)-2\frac{1-\mu(1-\beta)}{\sigma-1}\left(\frac{(1+\eta-\frac{1}{\sigma})(\frac{n}{2})^{\eta}+\psi\frac{\sigma-1}{\sigma}}{(\frac{n}{2})^{\eta}+1}\right)}\right) > 0, \tag{B.2}$$

where the sign of (B.2) follows immediately from (A.7). The function for the wage premium is given by

$$\frac{w_{h1}}{w_{l1}} = n_{h1}^{\eta-1/\sigma}n_{l1}^{1/\sigma} \implies \frac{d\frac{w_{h1}}{w_{l1}}}{dL_1} = (\eta+1/\sigma+\psi(1/\sigma))\frac{w_{h1}}{w_{l1}}\frac{dn_{h1}}{dL_1}.$$

Using the definition of ψ and rearranging terms yields

$$sgn\frac{d\frac{w_{h1}}{w_{l1}}}{dL_1} = sign(\eta/\theta-\rho/\sigma).$$

It is easily verified that

$$sgn\frac{d\frac{w_{h2}}{w_{l2}}}{dL_1} = -sgn\frac{d\frac{w_{h1}}{w_{l1}}}{dL_1}.$$

Turning to the relative land shares we have,

$$\frac{L_{h1}}{L_{l1}} = \frac{n_{h1}^{\eta-\frac{\sigma-1}{\sigma}}}{n_{l1}^{\frac{\sigma-1}{\sigma}}} \implies \frac{d\frac{L_{h1}}{L_{l1}}}{dL_1} = \left(\eta+\frac{\sigma-1}{\sigma}-\psi\frac{\sigma-1}{\sigma}\right)\frac{L_{h1}}{L_{l1}}\frac{dn_{h1}}{dL_1} > 0,$$

where the sign follows directly from the assumption that $\psi < 1$.

Turning to expected welfare levels note that

$$\frac{\partial V_{s1}}{\partial n_{h1}}|_{n_{h1}=1/2} = -\frac{\partial V_{s2}}{\partial n_{h1}}|_{n_{h1}=1/2}$$

It follows that the change in welfare from an increase in L_i is given by

$$\frac{\partial W_s}{\partial L_1}|_{n_{h1}=1/2} = \frac{1}{\theta}(V_{s1}+V_{s2})^{\frac{1}{\theta}-1}\frac{\partial V_{s1}}{\partial L_1}|_{n_{h1}=1/2} > 0$$

B.1 Extension to Multiple Regions and Skill Groups

In this subsection we show that our results presented above are similar when we expand the model beyond two regions and beyond two skill groups. We again consider the case where there is an increase in L_1. First, suppose there is J regions. Assuming that all regions are initially identical we can rewrite our spatial equilibrium conditions as

$$n_{li} = \frac{n_{hi}^{\psi}}{\sum_{i=1}^{J} n_{hi}^{\psi}} \forall i = 1, ..., J, \tag{B.3}$$

$$n_{h1}^{-1/\sigma} V_{h1} = n_{hi}^{-1/\sigma} V_{hi}, \forall i \neq 1 \tag{B.4}$$

$$W_s = \left(\sum_{i=1}^{J} V_{si}^{\theta} \right)^{1/\theta}, \tag{B.5}$$

$$\sum_{i=1}^{J} n_{hi} = n. \tag{B.6}$$

First note from the population constraint that

$$dn_{h1} = -\sum_{i=2}^{J} dn_{hi}. \tag{B.7}$$

Furthemore, around the symmetric equilibrium we will have $dn_{hi} = dn_{hj} \forall i, j \neq 1$. Around the symmetric equilibrium we have

$$\frac{\partial W_h}{\partial n_{h1}}|_{n_{h1}=n/J} = 0 \implies \frac{\partial V_{h1}}{\partial n_{h1}}|_{n_{h1}=n/J} = -\frac{\partial V_{hi}}{\partial n_{hi}}|_{n_{hi}=n/J} \forall i \neq 1. \tag{B.8}$$

Totally differentiating (B.4) with respect to L_1 and n_{hi} and combining with yields

$$n_{h1}^{1/\theta} \left(\frac{\partial V_{h1}}{\partial L_1}|_{n_{hi}=n/J} \right) dL_1 = J \left(\frac{\partial (n_{hi}^{1/\theta} V_{hi})}{\partial n_{hi}}|_{n_{hi}=n/J} \right) dn_{hi}. \tag{B.9}$$

Under our assumption that χ is sufficiently high we have from (B.9) that

$$\left(\frac{\partial (n_{hi}^{1/\theta} V_{hi})}{\partial n_{hi}}|_{n_{hi}=n/J} \right) < 0.$$

And given that

$$\left(\frac{\partial V_{h1}}{\partial L_1}|_{n_{hi}=n/J} \right) > 0,$$

it follows directly that

$$\frac{dn_{h1}}{dL_1} > 0, \quad \frac{dn_{hi}}{dL_1} < 0, \forall i \neq 1.$$

The key differences is that the impact on all regions $i \neq 1$ will be smaller as region 1 draws workers from multiple regions, taking less from each, than in the main text.

Now we consider the case when there are multiple skill groups. Specifically, redefine the skill index as $s = 1, ..., S$, with higher numbers corresponding to higher skills. Suppose that the production function is now given by

$$Y_i = A \left(\sum_s b_{si} n_{si}^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}} \tag{B.10}$$

Additionally, assume that production externalities are given by $b_{si} = n_{si}^{\eta_s}$ and that $\eta_{s+1} > \eta_s > \eta_{s-1} \forall s$ such that productive externalities fall with skill levels. Furthermore, suppose that residential externalities are also weaker for workers of lower skills such that $q_{si} = n_{si}^{\rho_s}$. Now notice that cost minimization implies that the relative wages between the skills levels 1 and S remain identical to to that of the model in the text when $\eta_S = \eta$. While for the skill levels s and $s + a$ the relative wages are given by

$$\frac{w_{(s+a)i}}{w_{si}} = \frac{n_{s+a,i}^{\eta_{s+a}}}{n_{si}^{\eta_s}} \left(\frac{n_{si}}{n_{s+a,i}} \right)^{1/\sigma}.$$

And the relative common welfare levels are given by

$$\frac{V_{s+a,i}}{V_{si}} = \frac{n_{s+a,i}^{(\eta_{s+a}+\rho_{s+a}-1/\sigma)}}{n_{si}^{(\eta_s+\rho_s-1/\sigma)}}. \tag{B.11}$$

Combining with the spatial equilibrium condition we can then write the number of any type s workers in region i as a function of the the number of highest skilled, type S workers.

$$n_{si} = \frac{n_{Si}^{\psi_s}}{\sum_i n_{Si}^{\psi_s}} n, \tag{B.12}$$

where

$$\psi_s = \frac{1/\sigma + 1/\theta - \rho_S - \eta_S}{1/\sigma + 1/\theta - \rho_s - \eta_s} \leq 1 \quad \forall s \in \{1, ..., S\}.$$

The sign follows directly from the assumption that $\eta_S > \eta_s$ and $\rho_S > \rho_s$ for any $s < S$. Therefore, If the term $1/\sigma + 1/\theta - \rho_S - \eta_S$ is positive, then any ψ_s will be positive as well. Suppose that this holds, then the analysis in section 2 can be used. Specifically, if $1/\sigma + 1/\theta - \rho_S - \eta_S$ is positive then an increase in developable land in region 1 will raise the number of all types of workers. Recalling, that around the symmetric equilibrium ψ_s is the elasticity of an increase in the number of type s workers in response to an increase in type S workers. Then, there will be a relatively larger increase in workers that have skill levels closer to the top skill level. Furthermore, while there would be an increase in inequality, the widening of the gap will be smaller for workers with higher skills.

Now suppose that $1/\sigma + 1/\theta - \rho_S - \eta_S < 0$. We now have to consider two cases. Specifically, we may have $1/\sigma + 1/\theta - \rho_s - \eta_s < 0$ for higher skill levels and $1/\sigma + 1/\theta - \rho_s - \eta_s > 0$ for lower skill levels. In this case we would see workers with higher skills move in tandem with the highest skilled workers toward city 1, in response to an increase in the supply of land. While some lower skilled workers would leave city 1 and migrate toward city 2. In this case we would see an increase in inequality between higher skilled workers that move toward city 1, while there would be a reduction in inequality between the highest and lowest skilled workers.

C Comparative Statics for Result 2

The comparative statics laid out here are very similar to those in Appendix B where ψ_a, ψ_d and ψ are now replaced with ζ_a, ζ_d and ζ. In addition, the exponent on the function for total income must be replaced from $(1-\mu(1-\beta)\sigma)/(\sigma-1)$ to $(1-\mu(1-\beta))/(\sigma-1)$. The equilibrium is now defined by

$$\frac{n_{h1}}{n_{h2}} = \left(\frac{V_{h1}}{V_{h2}}\right)^{\theta} \Longrightarrow$$

$$\left(\frac{n_{h1}}{n_{h2}}\right)^{\zeta_d-\zeta_a} = \left(\frac{A_1}{A_2}\right)^{1-\mu(1-\beta)} \left(\frac{n_{h2}+n_{l2}(n_{h1})}{n_{h1}+n_{l1}(n_{h1})}\right)^{\chi} \left(\frac{n_{h1}^{1+\eta-\frac{1}{\sigma}}+n_{l1}(n_{h1})^{\frac{\sigma-1}{\sigma}}}{(n_{h2}^{1+\eta-\frac{1}{\sigma}}+n_{l2}(n_{h1})^{\frac{\sigma-1}{\sigma}}}\right)^{\frac{1-\mu(1-\beta)}{\sigma-1}} \left(\frac{\bar{L}_1}{\bar{L}_2}\right)^{\mu(1-\beta)}. \tag{C.1}$$

The condition for the stability of the equilibrium is now

$$\chi(1+\zeta) > 2(\zeta_a-\zeta_d) + 2\left(\frac{1-\mu(1-\beta)}{\sigma-1}\right)\frac{\zeta\frac{\sigma-1}{\sigma}+(\eta+\frac{\sigma-1}{\sigma})\left(\frac{n}{2}\right)^{\eta}}{\left(\frac{n}{2}\right)^{\eta}+1}. \tag{C.2}$$

For further reference, we include the equations for the number of unskilled workers and the inequality measure,

$$n_{li} = \frac{\bar{L}_i^{\frac{\mu(1-\beta)}{\zeta_d}} n_{hi}^{\zeta}}{\bar{L}_i^{\frac{\mu(1-\beta)}{\zeta_d}} n_{hi}^{\zeta} + \bar{L}_j^{\frac{\mu(1-\beta)}{\zeta_d}} n_{hj}^{\zeta}} n, \quad \frac{W_h}{W_l} = \left(\frac{n}{\bar{L}_i^{\frac{\mu(1-\beta)}{\zeta_d}} n_{hi}^{\zeta} + \bar{L}_j^{\frac{\mu(1-\beta)}{\zeta_d}} n_{hj}^{\zeta}}\right)^{\zeta_d}. \tag{C.3}$$

Totally differentiating with respect to n_{h1}, $\bar{L}_{h1}$ and $\bar{L}_1$, evaluating the derivative at $n_{h1} = n/2$ and using the fact that $n'_{l1}(n_{h1})|_{n_{h1}=n/2} = \zeta$ yields

$$dn_{h1}|_{n_{h1}=n_{l1}=n/2} = \frac{\mu(1-\beta)n}{2}\left(\frac{\frac{d\bar{L}_{h1}}{\bar{L}_{h1}} - \frac{1}{2\zeta_d}\overbrace{\left(\chi - 2\frac{1-\mu(1-\beta)}{\sigma}\frac{1}{(\frac{n}{2})^{\eta}+1}\right)}^{(+)}\frac{d\bar{L}_1}{\bar{L}_1}}{\chi(1+\zeta)-2(\zeta_a-\zeta_d)-2\left(\frac{1-\mu(1-\beta)}{\sigma-1}\right)\frac{\zeta\frac{\sigma-1}{\sigma}+(\eta+\frac{\sigma-1}{\sigma})\left(\frac{n}{2}\right)^{\eta}}{\left(\frac{n}{2}\right)^{\eta}+1}}\right) \gtreqless 0.$$

The denominator is positive from (C.2). Given that $dL_{h1} \geq 0$ it follows that the sign of (C.4) will depend on the magnitude of $d\bar{L}_1/\bar{L}_1$. Note that we can write

$$\frac{d\bar{L}_1}{\bar{L}_1} = \frac{d\bar{L}_{l1}}{\bar{L}_{l1}} - \frac{d\bar{L}_{h1}}{\bar{L}_{h1}}.$$

Inserting into (C.4) yields

$$dn_{h1} \lesseqgtr 0 \iff \frac{d\bar{L}_{h1}}{L_{h1}} \lesseqgtr \omega \frac{d\bar{L}_{l1}}{L_{l1}}, \quad \omega \equiv \frac{\frac{1}{2\zeta_d}\left(\chi - 2\frac{1-\mu(1-\beta)}{\sigma}\frac{1}{(\frac{n}{2})^{\eta}+1}\right)}{1+\frac{1}{2\zeta_d}\left(\chi - 2\frac{1-\mu(1-\beta)}{\sigma}\frac{1}{(\frac{n}{2})^{\eta}+1}\right)} \in (0,1).$$

We can then write

$$\frac{dn_{l1}}{n_{l1}}|_{n_{s1}=n/2} = \zeta\frac{dn_{h1}}{n_{h1}} + \frac{\mu(1-\beta)}{2\zeta_d}\frac{d\bar{L}_1}{\bar{L}_1}.$$

Given that $d\bar{L}_1 > 0$, the second term is always positive. The first will be positive if $\zeta > 0$ and $dn_{h1} > 0$ or if $\zeta < 0$ and $dn_{h1} < 0$. When these conditions fail to hold, the results are ambiguous.

The skill premium is now

$$\frac{w_{h1}}{w_{l1}} = n_{h1}^{\eta-1/\sigma}n_{l1}^{1/\sigma} \implies \frac{d(\frac{w_{h1}}{w_{l1}})}{\frac{w_{h1}}{w_{l1}}} = (\eta - 1/\sigma)\frac{dn_{h1}}{n_{h1}} + (1/\sigma)\frac{dn_{l1}}{n_{l1}} = (\eta - \frac{1}{\sigma}(1-\zeta))\frac{dn_{h1}}{n_{h1}} + \frac{1}{\sigma}\frac{\mu(1-\beta)}{2\zeta_d}\frac{d\bar{L}_1}{\bar{L}_1}.$$

First, suppose that $d\bar{L}_1 = 0$ in which case $dn_{h1} > 0$ and $dn_{l1} = \zeta dn_{h1} > 0 \iff \zeta > 0$ and negative otherwise. Using the fact that $\zeta < 1$ yields

$$sgn\frac{d\frac{w_{h1}}{w_{l1}}}{\frac{w_{h1}}{w_{li}}} = sgn(\eta - \frac{1}{\sigma}(1-\zeta))\frac{dn_{h1}}{n_{h1}}.$$

Thus, when $dn_{h1} > 0$, the sign of the skill premium will depend on the sign of the term in brackets. It is easily verified that

$$sgn\frac{d\frac{w_{h2}}{w_{l2}}}{\frac{w_{h1}}{w_{l1}}} = -sgn\frac{d\frac{w_{h1}}{w_{l1}}}{\frac{w_{h1}}{w_{l1}}}.$$

Now, if $d\bar{L}_1 > 0$ then $dn_{h1} > 0$ and $dn_{l1} > 0$ if $d\bar{L}_{h1}/\bar{L}_{h1} > \omega d\bar{L}_{l1}/L_{l1}$ and $\zeta > 0$. Therefore, when $\eta > 1/\sigma(1-\zeta)$, the skill premium is rising. However, given that $dn_{h1} < 0$ if $d\bar{L}_{h1}/\bar{L}_{h1} < \omega d\bar{L}_{l1}/L_{l1}$, then the comparative statics are ambiguous.

For relative land rents, we have

$$\frac{r_{h1}}{r_{l1}} = \frac{n_{h1}^{\eta+\frac{\sigma-1}{\sigma}}}{n_{l1}^{\frac{\sigma-1}{\sigma}}}\bar{L}_1.$$

Thus, we have

$$sgn\left(\frac{d\frac{r_{h1}}{r_{h2}}}{\frac{r_{h1}}{r_{l1}}}\right) = (\eta + \frac{\sigma-1}{\sigma}(1-\zeta))\frac{dn_{h1}}{n_{h1}} + (1 - \frac{\sigma-1}{\sigma}\frac{\mu(1-\beta)}{2\zeta_d})\frac{d\bar{L}_1}{\bar{L}_1}.$$

The first term in brackets on the LHS is positive, given that $\zeta < 1$. Thus, when $dn_{h1} > 0$ and $d\bar{L}_1 = 0$, relative rents will rise. However, when $d\bar{L}_1 > 0$, the term depends on the magnitude of the second term in brackets which is always positive by inserting the definition of ζ_d. Thus, relative rents are always rising provided the number of skilled workers is rising. If the number of skilled workers is falling, the result is ambiguous.

Finally, for inequality we have

$$\frac{d\frac{W_h}{W_l}}{\frac{W_h}{W_l}} = -\frac{\mu(1-\beta)}{2}\frac{d\bar{L}_1}{\bar{L}_1} \leq 0,$$

which will be negative whenever $d\bar{L}_1 > 0$. Finally to consider welfare we have

$$\frac{\partial W_s}{\partial \bar{L}_{si}}|_{n_{h1}=1/2} = \frac{\mu(1-\beta)}{\theta}\left(V_{s1}+V_{s2}\right)^{\frac{1}{\theta}-1} V_{s1}\frac{d\bar{L}_{s1}}{\bar{L}_{s1}}|_{n_{h1}=1/2} > 0.$$

The sign follows from the assumption that $d\bar{L}_{s1} \geq 0$ from the text.

D Proof of Result 3

In this section we prove Result 3. We assume that the local city authority chooses the supply of land devoted to housing for each type of worker within each city, L_{si}, taken the land-use decision in the other city as exogenous. Given this choice, workers then make their location decisions. We solve the model via backwards induction by first solving for the level of social welfare taking L_{si} as given and then solving for the value of L_{si} that maximizes social welfare. Our focus is on a symmetric equilibrium such that $n_{si} = n/2$ and $L_{s1} = L_{s2}$ for $s = \{h, l\}$. To begin, for reference we have the welfare index defined as

$$W_s \equiv \left(V_{s1}^{\theta} + V_{s2}^{\theta}\right)^{\frac{1}{\theta}}, \tag{D.1}$$

and the component of welfare given the zoning requirements is given by

$$\begin{aligned} V_{hi} &= \kappa A_i^{1-\mu(1-\beta)}\frac{n_{hi}^{\rho+\eta(1-\mu(1-\beta))-(\frac{1}{\sigma}+\mu(1-\beta)(1-\frac{1}{\sigma}))}}{(n_{hi}+n_{li})^{\chi}}\left(n_{hi}^{\eta+\frac{\sigma-1}{\sigma}}+n_{li}^{\frac{\sigma-1}{\sigma}}\right)^{\frac{1-\mu(1-\beta)}{\sigma-1}}(L_{hi})^{\mu(1-\beta)}, \\ V_{li} &= \kappa A_i^{1-\mu(1-\beta)}\frac{n_{li}^{-(\frac{1}{\sigma}+\mu(1-\beta)(1-\frac{1}{\sigma}))}}{(n_{hi}+n_{li})^{\chi}}\left(n_{hi}^{\eta+\frac{\sigma-1}{\sigma}}+n_{li}^{\frac{\sigma-1}{\sigma}}\right)^{\frac{1-\mu(1-\beta)}{\sigma-1}}(L_{li})^{\mu(1-\beta)}. \end{aligned} \tag{D.2}$$

Thus, the problem for the local planner is

$$\max_{L_{si}} \frac{1}{1-\epsilon}\left(\sum_s n_{si}\left(\left(V_{s1}^{\theta}+V_{s2}^{\theta}\right)^{\frac{1}{\theta}}\right)^{1-\epsilon}\right). \tag{D.3}$$

Inserting $L_{li} = L_i - L_{hi}$ into (D.2) allows us to treat (D.3) as a single variable maximization problem. The first-order condition with respect to L_{hi} is given by

$$\mu(1-\beta)\left((V_{h1}^{\theta}+V_{h2}^{\theta})^{\frac{1-\epsilon}{\theta}-1}\frac{V_{hi}^{\theta}}{L_{hi}} - (V_{l1}^{\theta}+V_{l2}^{\theta})^{\frac{1-\epsilon}{\theta}-1}\frac{V_{li}^{\theta}}{L_{li}}\right) = 0. \tag{D.4}$$

In a symmetric equilibrium $V_{s1} = V_{s2}$, thus, (D.4), after some manipulation, collapses to

$$\left(\frac{V_{hi}}{V_{li}}\right)^{1-\epsilon} = \frac{L_{hi}}{L_{li}}.$$

Combining with (D.2) yields the solution in Result 3. The second-order condition requires that the second derivate of (D.3) when evaluated at the solution be negative. This can be written after some manipulation and combining like terms as

$$\mu(1-\beta)\left((1-\epsilon-\theta)2^{\frac{1-\epsilon}{\theta}-2}\left(\sum_s \frac{V_{si}^{1-\epsilon}}{L_{si}^2}\right) + \theta(\mu(1-\beta)-1)2^{\frac{1-\epsilon}{\theta}-1}\left(\sum_s \frac{V_{si}^{1-\epsilon}}{L_{si}^2}\right)\right) < 0.$$

The sign follows from the fact that both the term $(1-\epsilon-\theta)$ is negative, given that $\theta > 1$, and $\mu(1-\beta) < 1$.

E Derivations in Section 6

In this section we derive the results presented in Section 6 of the text. The problem of a homeowner is given by

$$U_{si}^O = \delta u_i(n_{hi}, n_{li}) q_{si}(n_{hi}, n_{li}) \left(\frac{h_{si}^O}{\mu}\right)^{\mu} \left(\frac{x_{si}^O}{1-\mu}\right)^{1-\mu} \epsilon_i$$
$$s.t. \quad w_{si} = m_s p_i h_{si}^O + x_{si}^O + c_{si}(n_{si}^O). \tag{E.1}$$

The demand functions and deterministic indirect utility are then

$$h_{si}^O = \mu \frac{w_{si} - c_{si}(n_{si}^O)}{m_s p_i}, \quad x_{si}^O = (1-\mu)(w_{si} - c_{si}(n_{si}^O)), \quad V_{si}^O(\epsilon_i) = \delta u_i(n_{hi}, n_{li}) q_{si}(n_{hi}, n_{li}) \frac{w_{si} - c_{si}(n_{si}^O)}{(m_s p_i)^{\mu}} \epsilon_i. \tag{E.2}$$

Equating the indirect utility of a homeowner with that of a renter from (3) yields the function (26) in the text. For a city to have both renters and homeowners among both types of workers in a symmetric equilibrium we require that

$$0 < n_{si}^O = \left(\frac{(\delta - m_s^{\mu}) w_{si}}{Q}\right)^{1/\gamma} < n/2 \tag{E.3}$$

Rewritten in terms of the wages of each type when evaluated at $n_{hi} = n_{li} = n/2$ we have

$$0 < \frac{n}{2}^{(\eta - 1/\sigma)}((\frac{n}{2})^{\eta + \frac{\sigma-1}{\sigma}} + (\frac{n}{2})^{\frac{\sigma-1}{\sigma}})^{\frac{1}{\sigma-1}} < (\frac{n}{2})^{\gamma} \frac{Q}{\delta - m_h^{\mu}}, \tag{E.4}$$

$$0 < \frac{n}{2}^{-1/\sigma}((\frac{n}{2})^{\eta + \frac{\sigma-1}{\sigma}} + (\frac{n}{2})^{\frac{\sigma-1}{\sigma}})^{\frac{1}{\sigma-1}} < (\frac{n}{2})^{\gamma} \frac{Q}{\delta - m_l^{\mu}}, \tag{E.5}$$

where the top row corresponds to the condition for skilled workers and the bottom row for unskilled workers. This conditions will hold provided Q is sufficiently high and we assume this condition is met throughout the analysis. To provide some intuition, consider the special case where $n = 2$ in which case the condition becomes $Q > 2^{1/(\sigma-1)}(\delta - m_s^{\mu})$. The parameter Q represents the additional costs to homeownership when there are no negative externalities from other homeowners. While $\delta - m_s^{\mu}$ captures net capital gains per unit of housing. Thus the homeownership costs must exceed a multiple of the net capital gains. Using the same method as above but taking into account the differences in costs by racial group, as set out in Section 6.2, the relationship in (27) follows immediately.

E.1 The Impact of Land-Use Regulation on Wages

Here we consider when wages will rise or fall in response to an increase in L_1. Using (15) it is straightforward to show that

$$\frac{\partial n_{l1}}{\partial n_{h1}}|_{n_{h1}=n/2} = \psi.$$

And given that $dn_{h1}/dL_1 > 0$, we can then write the signs of the impact of an increase in L_1 on wages as

$$sgn\left(\frac{dw_{l1}}{dL_1}\right)|_{[n_{h1}=n/2]} = sgn\left(\frac{\eta}{\sigma-1}\left(\frac{n}{2}\right)^{\eta} - \frac{1}{\sigma}(1-\psi)\right), \tag{E.6}$$

$$sgn\left(\frac{dw_{h1}}{dL_1}\right)|_{[n_{h1}=n/2]} = sgn\left(\frac{\eta\sigma}{\sigma-1}\left(\frac{n}{2}\right)^{\eta} + \eta - \frac{1}{\sigma}(1-\psi)\right). \tag{E.7}$$

These terms will more likely be positive when η is high and the total number of skilled workers, n, is large. Furthermore, wages for skilled workers are more likely to rise than unskilled workers.

References

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