Water for Arid Regions: An Economic Geography Approach *

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Abstract

A primary motivation for trade is to ameliorate the uneven distribution of resources across space. Asymmetries in productive land, natural resources and labor supply all contribute to the type and location of economic activity across regions. This paper considers a very specific problem.
1 Introduction

Historically, a local supply of water has been a crucial component for the location of cities and agriculture. Indeed, an alternate history of westward expansion of the United States in the 19th century focuses on the Bureau of Land Reclamation and the Army Corps of Engineers struggling to reconfigure existing water resources to accommodate the idiosyncratic land use patterns of the early settlers (Reisner, 1986). Increasing urbanization has placed considerable stress on existing water resources in arid regions. In response, many regions import water via interbasin transfers to supplement existing resources. Recent research suggests that water transportation infrastructure has greatly reduced the number of water-stressed cities globally (Olmstead (2010), McDonald, et al. (2014)). Such infrastructure projects can be attractive for regional governments looking to promote growth. For instance, in the 1960’s, California governor Pat Brown, inaugurated the State Water Project, which was developed to supply Southern California cities and agriculture with water from the Northern part of the state. His stated intention was to “correct an accident of people and geography” (National Geographic, 2010). Since the project was begun there has been a significant increase in the population of the Southern California region as evidenced in Table [1]. While the San Francisco area grew by roughly 60%, the population of Los Angeles increased threefold. However, most striking is the rise of Southern California’s formerly suburban cities such as San Diego, who saw an increase of nearly 700% from their 1950 population, and the Riverside and San Bernardino region whose population increased fourteenfold over the time period.

Table 1: Urban Population in California Cities in 1950 and 2010 in 1000’s.

<table>
<thead>
<tr>
<th>Northern California Cities</th>
<th>1950</th>
<th>2010</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sacramento</td>
<td>212</td>
<td>1,724</td>
</tr>
<tr>
<td>San Francisco–Oakland</td>
<td>2,022</td>
<td>3,281</td>
</tr>
<tr>
<td>San Jose</td>
<td>176</td>
<td>1664</td>
</tr>
<tr>
<td>Southern California Cities</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Los Angeles</td>
<td>3,997</td>
<td>12,151</td>
</tr>
<tr>
<td>Riverside–San Bernardino</td>
<td>136</td>
<td>1,933</td>
</tr>
<tr>
<td>San Diego</td>
<td>433</td>
<td>2,957</td>
</tr>
</tbody>
</table>

Source: Demographia (2014)

Recent empirical work has shown that households are increasingly moving towards warmer and drier climates, in particular in the US Southwest, the Pacific coast and the South Atlantic
This paper contends that a similar outcome is taking place with the location of agricultural production. In order to motivate the argument, Figure [1] plots the share of US population and US agricultural output by region from 1960 to 2000. Two features stand out from these graphs. The first is that in general the population is growing in regions with more temperate climates in the US South, Southwest, Pacific and Mountain Regions. Second, the share of agricultural production by region tends to move in tandem with the population which suggests that local agriculture, is to some extent, reliant on a local population as a source of demand.

The bottom left chart in Figure [1] plots the average total factor productivity (TFP) for agriculture by region. Given that TFP measures the state of technology in any given time period, we would expect to see the magnitudes to be largely uniform among homogeneous regions, provided they all have access to the same technology. And in fact, the majority of regions appear to show roughly the same growth in TFP over the period.
Figure 1: US Population, Agricultural Output, Agricultural Total Factor Productivity and Land Values

† Census Regions: New England-MA, ME, NH, VT, RI, CT, Middle Atlantic-NY, NJ, PA, East North Central-OH, IN, IL, MI, WI, West North Central-MN, IA, MO, ND, SD, NE, KS, South Atlantic-DE, MD, VA, WV, NC, SC, GA, FL, East South Central-KY, TN, AL, MS, West South Central-AR, LA, OK, TX, Mountain-MT, ID, WY, CO, NM, AZ, UT, NV, Pacific-WA, CA, OR. Alaska, Hawaii, Puerto Rico and District of Columbia are omitted to maintain continuity between data sets.
Figure 2: Agricultural Total Factor Productivity by Region, 1960-2004

Figure 3: Land Values by Region, 1975-2010
However, TFP in the South Atlantic and the Pacific Regions, is well above the remainder of the country. When California is isolated, in some years the state’s TFP is more than twice that of the least productive regions. Finally, the bottom right graph in Figure[1] plots regional land values from 1975-2010, where the regions that generate the highest prices are along the pacific and the Eastern seaboard. Again, isolating California, its land values are significantly above the rest of the nation.

This paper develops a model to investigate water use patterns across regions when the physical endowments of land varies across space. The model consists of two regions of equal size each of which devote land to either agricultural production or to housing for residents who work in an urban manufacturing sector. The regions are separated by an uninhabitable valley. There is a fixed number of households who gain utility from land, agricultural goods, water and region specific natural amenities. In addition, there is a fixed quantity of water that is located solely in one region. A publicly financed water distribution network is developed to transit water across both regions for urban and agricultural use. Each city produces a manufacturing good which is freely traded across regions, and the manufacturing sector in each region shows increasing returns to scale dependent on the size of the regional labor force. Agricultural land in each region differ in productivity, and there are iceberg transport costs associated with the trade of the agricultural good. Households distribute themselves across regions so that a spatial equilibrium is found when utility equalizes across regions. This paper looks at a specific subset of possibilities of this framework, namely, the case where one region is endowed with both a more productive agricultural sector and higher level of natural amenities yet is devoid water resources. Three trade regimes are considered which are dependent on the level of the agricultural productivity differential between both regions. An autarkic regime where each region produces the agricultural good solely for the local population. A supplemental regime, whereby both regions produce the agricultural good, and the more productive region supplements the supply of the other region. Finally, the case of the more productive region specializing in the production of the agricultural good.

It is found that agricultural productivity acts as an agglomerative force, as household’s benefit from allowing the more productive land to be used for agriculture, leading to concentration in the other region. However, increases in transport costs or natural amenities of the more productive region defuse the agglomerative effect of the agricultural productivity. Economies of scale are found to have little effect when the population is more evenly dispersed, however,
increases in agglomeration economies when the population share differential is high increases concentration towards the larger region.

The contribution of this paper is to explore how the introduction of water transportation infrastructure in a two region model with regional asymmetries in agricultural and manufacturing production, resources and natural amenities, will affect the share of population between regions and the distribution of land between agricultural and urban use within regions. From, a policy perspective this paper develops a novel framework for conceptualizing the efficient allocation of water resources across space. In addition, it allows for the cost-benefit analysis of water distribution infrastructure and the comparison of various pricing and tax financing schemes. This paper, continues in a long tradition of using computable general equilibrium (CGE) models of the monocentric city to explore the effects of policy changes including transportation costs on land rents and congestion (Arnott and Mackinnon (1977), Arnott and Mackinnon (1978), Tikoudis et. al. (2015)), the development of urban subcenters (Sullivan (1986), Helsley and Sullivan (1991)) and urban environmental policy and land use (Verhoef and Nijkamp (2002), Bento et. al. (2006)).

From a theoretical point of view the model combines the closed monocentric city model (Pines and Sadka (1984)) and the two region New Economic Geography (NEG) literature pioneered by Krugman (1991) and Fujita et. al. (1999). The model is closed in the sense that the total population is fixed, household utility is endogenous and rental income is redistributed back to households. This allows for welfare analysis under various trade and policy regimes. In addition, this model encloses the space of the model by fixing the quantity of land reflecting the fact that land use is limited by physical or political boundaries. Tabuchi (1997) integrated the Alonso-Muth-Mills model in to the NEG framework, however, his model retained a central tenet of the monocentric city that agricultural land rent is exogenous. In contrast, this model, by fixing the quantity of land in each region that can be used for urban or agricultural endogenizes land rent at the boundary of the city, creating a tension between urban agglomerative processes and increasing agricultural productivity, reinforcing Pflüger and Tabuchi’s statement of “the long standing wisdom in spatial economics that ultimately there is only one immobile resource, land” (2010). Other authors have explored the effect of limits to developable land on urban growth (Helpman (1998), Saiz (2010), Chatterjee and Eyigungor (2012)) however the the effect of heterogeneity in the agricultural productivity as a factor in the urban land supply constraints is not treated. Picard and Zheng (2005) extend the model of Ottaviano,
et. al. (2002) to integrate more explicitly an agricultural sector with transport costs, who compete with the manufacturing sector for labor. This model, on the other hand, assumes that agriculture competes with the urban households for land and water. Matsuyama (1992) proposed a endogenous growth model, in which he considered both a closed an a small open economy. He found a positive link between agricultural productivity and growth in a closed economy and a negative link in an open setting. The analysis in this seems to confirm this result. Under autarky, the more productive region has a larger share of the population and thus a larger manufacturing sector as well as more abundance in the agricultural good. While, when trade is possible, the region with less productive agriculture has a larger manufacturing sector. Recent research has also focused on how a limited land supply can dampen agglomerative forces (Pflüger and Südekum (2007), Pflüger and Tabuchi (2010)). In our model, this result occurs due to the tension between household’s preferences for natural amenities and agricultural goods. When a region is abundant in natural amenities, households are willing to pay a higher price to live in that location. This ultimately increases land rents and diminishes the productivity effect on agricultural prices, which further reduces a households willingness to locate in another region; In which case, households are more evenly split across regions. However, when natural amenities are low the agricultural productivity effect will dominate, driving households to the other region, which is reinforced by the agglomeration economies as one region becomes relatively more populous than the other. This is consistent with quality of life and urban amenities literature (Roback (1982), Rappaport (2008)). As in, (Pflüger and Südekum (2007), an intuitively appealing outcome of this model is that unlike the standard NEG model, where at a critical value the whole population goes instantaneously from dispersion to agglomeration, this model shows gradual shifts in the population with changes in productivity. Finally, the model is novel in introducing water and the interregional transportation infrastructure into the monocentric city and NEG models.

The remainder of the paper is presented as follows. Section 2 outlines the model and describes the equilibrium under various trade regimes. Section 3 analyzes the model using specific functional forms. Section 4 describes the functional forms and parameter values which were used to calibrate the numerical simulation. Section 6 presents and discusses the numerical results. Section 7 proposes topics and extensions for future research. Finally, section 8 concludes.
2 The Model

2.1 Model Overview

This section considers the size and location of manufacturing and agricultural production in a two region spatial model when water is a mobile factor across regions.

Table [2] provides a notational glossary.

The model consists of a small country populated by \( N \) identical households. The country is divided into two regions, named 1 and 2, respectively, with \( \lambda N \) households in region 1 and \((1 - \lambda)N\) in region 2. The space of the country is a line of length \( 2L + L_s \), where \( L \) is the size of region \( i = 1, 2 \) and \( L_s \) is a length separating the two regions. The land and water supply of the country are commonly owned by all residents. Each region employs a share of the population to produce a manufacturing good in the central business district (cbd) of a monocentric city located in each region. A share of each region’s land is used for housing the local population.

The remainder of the land is devoted to agricultural production. Demand for land by households is fixed at a single unit which are chosen such that \( L = N \). This implies that the size of the city in region 1 is \( \lambda N \) and in region 2 is \((1 - \lambda)N\). While, symmetrically the land devoted to agriculture in region 1 is \((1 - \lambda)N\) and in region 2 is \( \lambda N \). The country contains a fixed supply water, \( W \), which is located at the cbd of region 2 and is used for irrigation by the agricultural sector and by households for personal consumption. The supply of water is assumed to be fully allocated. A publicly financed infrastructure network transports water from the source to households and the agricultural sector in each region. Figure [2] gives a visual description of the space of the model.

![Figure 4: Regional Space](image)

The top line gives the land distribution. In the center is the length \( L_s \) that separates the two regions. At the boundary of \( L_s \) and region 1 and 2 is the local cbd. Along the distance \( x_i \) is the length of the city which ends at \( \lambda N \) for region 1 and \((1 - \lambda)N\) for region 2. The remaining
Table 2: Notational Glossary

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a_i$</td>
<td>household demand for agricultural good in region $i$</td>
</tr>
<tr>
<td>$i$</td>
<td>regional subscript</td>
</tr>
<tr>
<td>$m_i$</td>
<td>household demand for manufacturing good</td>
</tr>
<tr>
<td>$p^a_i$</td>
<td>regional agricultural good price</td>
</tr>
<tr>
<td>$p^{ma}_i$</td>
<td>regional manufacturing price, numeraire</td>
</tr>
<tr>
<td>$p^w$</td>
<td>common water price</td>
</tr>
<tr>
<td>$r^a_i$</td>
<td>regional agricultural land rent</td>
</tr>
<tr>
<td>$r_i(x_i)$</td>
<td>regional bid-rent function</td>
</tr>
<tr>
<td>$\bar{r}$</td>
<td>rental transfer</td>
</tr>
<tr>
<td>$t$</td>
<td>household commuting cost</td>
</tr>
<tr>
<td>$w^a_i$</td>
<td>agricultural water demand per unit of land</td>
</tr>
<tr>
<td>$w^u_i$</td>
<td>household water demand</td>
</tr>
<tr>
<td>$\bar{w}$</td>
<td>per-capita supply of water</td>
</tr>
<tr>
<td>$x_i$</td>
<td>distance from the cbd</td>
</tr>
<tr>
<td>$y_i$</td>
<td>regional wage</td>
</tr>
<tr>
<td>$W^a_i$</td>
<td>total regional agricultural water demand</td>
</tr>
<tr>
<td>$L^a_i$</td>
<td>total regional agricultural land</td>
</tr>
<tr>
<td>$A$</td>
<td>shift factor on marginal product of labor</td>
</tr>
<tr>
<td>$A_i$</td>
<td>regional marginal product of labor</td>
</tr>
<tr>
<td>$I_i$</td>
<td>regional net income</td>
</tr>
<tr>
<td>$L$</td>
<td>total land in each region</td>
</tr>
<tr>
<td>$L_s$</td>
<td>distance between regions</td>
</tr>
<tr>
<td>$N$</td>
<td>total population</td>
</tr>
<tr>
<td>$T$</td>
<td>ratio of net incomes in supplemental and specialized regimes</td>
</tr>
<tr>
<td>$U_i$</td>
<td>regional utility level</td>
</tr>
<tr>
<td>$W$</td>
<td>available supply of water</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>water share of agricultural production costs</td>
</tr>
<tr>
<td>$\beta_i$</td>
<td>regional agricultural productivity</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>budget share of water</td>
</tr>
<tr>
<td>$\delta$</td>
<td>degree of economies of scale in manufacturing sector</td>
</tr>
<tr>
<td>$\eta$</td>
<td>budget share of manufacturing goods</td>
</tr>
<tr>
<td>$\theta$</td>
<td>agricultural water subsidy</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>share of households in region 1</td>
</tr>
<tr>
<td>$\mu$</td>
<td>budget share of agricultural goods</td>
</tr>
<tr>
<td>$\rho$</td>
<td>defines the elasticity of substitution in agricultural production</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>elasticity of substitution between land and water in agricultural production</td>
</tr>
<tr>
<td>$\tau$</td>
<td>agricultural transport costs across regions</td>
</tr>
<tr>
<td>$\phi_i$</td>
<td>shift parameter denoting household preferences for regions</td>
</tr>
<tr>
<td>$\Theta$</td>
<td>functional abbreviation</td>
</tr>
<tr>
<td>$\Lambda$</td>
<td>functional abbreviation</td>
</tr>
<tr>
<td>$\Phi$</td>
<td>ratio of net incomes in autarkic equilibrium</td>
</tr>
</tbody>
</table>
land up to the length $N$ in each region is devoted to agriculture and is denoted in the figure by $Ag1$ and $Ag2$, respectively. The second line describes the infrastructure, denoted as “Pipe” in the figure. The water supply is located at the cbd of region 2. It travels across the length $L$ of region to the right, while to the left it travels the length $L + L_s$ in order to supply region 1.

2.2 Demand

2.2.1 Households

Each household supplies labor inelastically to the manufacturing sector for which they receive the wage $y_i$. They each work at the cbd and live some distance $x_i$ from the cbd, where they face the land rents $r_i(x_i)$ and commuting costs $t(x_i)$. In addition to wage income, households receive a transfer $\bar{r}$, which is the household share of aggregate land rents, and face the flat tax $f$, which is used to finance the water distribution infrastructure. Household’s have preferences over the numeraire manufacturing good, $m_i$, urban water, $w_u^i$ and the agricultural good, $a_i$, for which they face the agricultural price $p^a_i$ and the common water price $p^w$, and $p^m$ is the numeraire and equal to 1. The utility maximization problem is then given by,

$$\max_{m_i, a_i, w_u^i} U_i(m_i, a_i, w_u^i, \phi_i)$$

s.t.

$$y_i + \bar{r} - f - r_i(x) - t(x) = m_i + p^a_i a_i + p^w_i w_u^i, \quad i = 1, 2,$$

where $\phi_i$ is a shift factor that measures region specific natural amenities such as weather or attractive landscape. Solving for $m_i$ from the budget constraint and inserting it into the utility function, the first order condition yields,

$$p^w = \frac{U_i, w_u^i}{U_i, m_i}, \quad p^a_i = \frac{U_i, a_i}{U_i, m_i},$$

12
where the second subscript denote partial derivatives. Combining [] with the budget constraint yields the following household uncompensated demand functions.

\[
m_i = m_i(y_i + \bar{r} - f - r_i(x_i) - t(x_i), p^a_i, p^w_i; \phi_i),
\]

(3)

\[
a_i = a_i(y_i + \bar{r} - f - r_i(x_i) - t(x_i), p^a_i, p^w_i; \phi_i),
\]

(4)

\[
w^n_i = w^n_i(y_i + \bar{r} - f - r_i(x_i) - t(x_i), p^a_i, p^w_i; \phi_i).
\]

(5)

The indirect utility function is then given by

\[
V_i(y_i + \bar{r} - f - r_i(x_i) - t(x_i), p^a_i, p^w_i; \phi_i).
\]

(6)

For household’s to be indifferent across locations in the city implies that the derivative of the indirect utility function with respect to \( x_i \) be zero, which yields,

\[
r'_i(x_i) = -t'(x_i)
\]

(7)

Integrating over \( x_i \) gives,

\[
r_i(x_i) = -t(x_i) + k,
\]

(8)

where \( k \) is a constant of integration. Using the terminal condition that the rent at the boundary of the city equal the agricultural rent \( r^a_i \), gives the bid-rent function for each region,

\[
r_1(x_1) = r^a_1 + t(\lambda N) - t(x_1), \quad x_1 \in [0, \lambda N]
\]

(9)

\[
r_2(x_2) = r^a_2 + t(1 - \lambda)N - t(x_2), \quad x_2 \in [0, (1 - \lambda)N]
\]

(10)

where use is made of the fact that the boundary of the city in region 1 is \( \lambda N \) and \( (1 - \lambda)N \) in region 2.
2.3 Supply

2.3.1 Manufacturing

The manufacturing good is produced by a continuum of small firms, with a linear technology utilizing solely labor. Producers face the wage cost \( y_i \). The aggregate profit function for manufacturing firms in each region is given by,

\[
A_1 \lambda N - y_1 \lambda N, \tag{11}
\]

\[
A_2 (1 - \lambda) N - y_2 (1 - \lambda) N \tag{12}
\]

where \( A_i \) is the marginal product of labor and are taken is given by firms. The industry is assumed to exhibit increasing returns from population size due to agglomeration economies at the aggregate level, which are captured in each region by the term \( A_1 = A_1(\lambda N; \delta), A_2 = A_2((1 - \lambda)N; \delta) \). At the firm level, perfect competition drives profit to zero yielding,

\[
A_1(\lambda N; \delta) = y_1, \tag{13}
\]

\[
A_2((1 - \lambda)N; \delta) = y_2. \tag{14}
\]

where \( \delta \) is a parameter relating the elasticity of the marginal product of labor to local population size.

2.4 Agricultural Production

Agriculture is organized competitively. It is produced using water, \( W_{ai} \) and land \( L_{ai} \), with the linearly homogeneous production function \( F_i(W_{ai}^a, L_{ai}^a; \beta_i) \), where \( \beta_i \) is a region specific shift factor capturing the productivity of agriculture. It is assumed that \( \beta_1 \geq \beta_2 \). Given that in equilibrium the land devoted to agriculture in each region is simply the share not used by households, it is useful to write the production function in intensive form as,

\[
F_i(W_{ai}^a, L_{ai}^a; \beta_i) = L_{ai}^a f_i\left(\frac{W_{ai}^a}{L_{ai}^a}, 1; \beta_i\right) = L_{ai}^a f_i(w_{ai}^a; \beta_i). \tag{15}
\]
Producers face the prices of water, \( p^w \), and land rent \( r^a \), and charge the price \( p^a_i \). The profit function for unit of land is then,

\[
(16) \quad p^a_i f_i(w^a_i; \beta_i) - p^w w^a_i - r^a_i
\]

The first-order condition is given by,

\[
(17) \quad p^a_i f_i'(w^a_i; \beta_i) - p^w = 0,
\]

which yields the agricultural water demand function per unit of land,

\[
(18) \quad w^a_i(p^a_i, p^w; \beta_i).
\]

Finally, agricultural rents adjust until profits are equal to zero,

\[
(19) \quad p^a_i f_i(w^a_i(p^a_i, p^w; \beta_i); \beta_i) - p^w w^a_i(p^a_i, p^w; \beta_i) = r^a_i.
\]

2.5 Government

The government plays two roles. First, it collects land rents and redistributes the proceeds back to residents as the lump sum transfer \( \bar{r} \). Second, it oversees the construction of the water transportation infrastructure and the pricing of water, and levies a tax on households for any additional costs not covered by the sale of water, \( f \). Note that the assumption of a common flat tax for residents of both regions ensures that there is no migration by households looking to benefit from preferential tax rates.

2.5.1 Rental Transfer

Integrating over the household bid-rent functions yields,

\[
(20) \quad \int_0^{\lambda N} r_1(x_1)dx_1 = \lambda N r^a_1 + \int_0^{\lambda N} t(x_1)dx_1, \quad \int_0^{(1-\lambda) N} r_2(x_2)dx_2 = (1-\lambda) N r^a_2 + \int_0^{(1-\lambda) N} t(x_2)dx_2.
\]
The rental revenue from agriculture in region 1 and 2, respectively, is \((1 - \lambda)Nr_1^a\), and \(\lambda Nr_2^a\). It follows that the rental transfer is given by,

\[
\bar{r} = r_1^a + r_2^a + \int_0^{\lambda N} t(x_1)dx_1 + \int_0^{(1-\lambda)N} t(x_2)dx_2.
\]

\[\text{(21)}\]

### 2.5.2 Infrastructure Tax

The manufacturing good is used to produce the water transportation infrastructure. The size of the infrastructure is modeled as proportional to the share of the total water supply going in each direction from the source to region 1 or 2, multiplied by the distance the water must travel. The infrastructure needed to supply each region with water is then,

\[\text{Region 1 : } (L + L_s) \times \left[\frac{(1 - \lambda)Nw_1^a + \lambda Nw_1^v}{W}\right], \quad \text{Region 2 : } L \times \left[\frac{\lambda Nw_2^a + (1 - \lambda)Nw_2^v}{W}\right].\]

\[\text{(22)}\]

By assumption the water is fully allocated so the total infrastructure can be rewritten as,

\[\text{Total : } L + L_s\left[\frac{(1 - \lambda)Nw_1^a + \lambda Nw_1^v}{W}\right].\]

\[\text{(23)}\]

The total revenue from the sale of water is simply \(p^wW\), thus the per-capita infrastructure tax can be written as,

\[f = 1 + L_s\left[\frac{(1 - \lambda)w_1^a + \lambda w_1^v}{W}\right] - p^w \bar{w},\]

\[\text{(24)}\]

where use is made of the fact that \(L = N\) and \(\bar{w} = \frac{W}{N}\) is the share of available water per person. The central term on the right is the region 1 water share of the total which must to be transported the additional length \(L_s\). It is useful, in order to simplify notation, to define the household net income as,
\[ I_1 \equiv y_1 + r^a_2 + p^w \bar{w} + \int_0^{(1-\lambda)N} t(x_1) dx_1 + \int_0^{(1-\lambda)N} t(x_2) dx_2 - t(\lambda N) \]
\[ \quad - [1 + L_s(\frac{(1-\lambda)Nw^a_1 + \lambda Nw^u_1}{W})], \]
\[ I_2 \equiv y_2 + r^a_1 + p^w \bar{w} + \int_0^{(1-\lambda)N} t(x_1) dx_1 + \int_0^{(1-\lambda)N} t(x_2) dx_2 - t(1-\lambda)N \]
\[ \quad - [1 + L_s(\frac{(1-\lambda)Nw^a_1 + \lambda Nw^u_1}{W})]. \]

### 2.6 Transport Costs and Equilibrium

An additional feature of the model are transportation costs for the agricultural good, which are assumed to take the Samuelson iceberg form and are represented by the parameter \( \tau \geq 1 \). The assumption is that in transit a share of the transported good is lost, so in order to receive 1 unit of the good, \( \tau \) units must be ordered, with the share \( \tau - 1 \) vanishing in transit. Therefore, in order for an agricultural producer in region 1 to sell to a consumer in region 2, she must set a price \( p^a_2 = \tau p^a_1 \). Given the asymmetries in the location of water and manufacturing and agricultural productivity, it is of interest how the population and thus manufacturing and agricultural production will be distributed across the two regions. *A priori* it is not possible to know in which direction the trade will flow. However, given the assumption that the agricultural sector in region 1 is more productive, the analysis will focus on trade from region 1 to 2 as productivity increases. We consider three possible regimes: autarky, supplemental, partial specialization.

#### 2.6.1 Autarky

An autarkic equilibrium will occur when \( \tau \) is sufficiently high such that there is no trade between regions. In which case, each region produces agriculture solely for the local population. The regional agricultural goods equilibrium is then,

\[ (1-\lambda)Np^a_1 f_1(w^a_1(p^a_1, p^w_1); \beta_1) = \lambda Na_1(I_1, p^a_1, p^w; \phi_1), \]
\[ \lambda Np^a_2 f_2(w^a_2(p^a_2, p^w_2); \beta_2) = (1-\lambda)Na_2(I_2, p^a_2, p^w; \phi_2). \]
These equations will yield the equilibrium agricultural price for each region. The government sets the water price to clear the market. The equilibrium condition is,

\[ W = \lambda N w_1^a(I_1, p_1^a, p^w; \phi_1) + (1-\lambda) N w_2^a(I_2, p_2^a, p^w; \phi_2) + (1-\lambda) N w_1^a(p_1^a, p^w; \beta_1) + \lambda N w_2^a(p_2^a, p^w; \beta_2) \]

Provided these two markets are in equilibrium, by Walras Law the manufacturing sector will also be in equilibrium.

### 2.6.2 Supplemental

A supplemental trade equilibrium occurs when both regions produce agriculture, but one region produces in excess of local demand and trades the remaining share to supplement the other region. I will focus on the case where \( \beta_1 \) is sufficiently high such that trade flows from region 1 to 2. In order for trade to occur, the imported price must be no higher than the local price. If both regions are producing this implies that \( p_2^a = \tau p_1^a \). Therefore, market clearing in agriculture is simply that aggregate supply equal aggregate demand,

\[ (1-\lambda) N p_1^a f_1(w_1^a(p_1^a, p^w; \beta_1); \beta_1) + \lambda N p_2^a f_2(w_2^a(p_2^a, p^w; \beta_2); \beta_2) = (1-\lambda) N a_1(I_1, p_1^a, p^w; \phi_1) + (1-\lambda) N a_2(I_2, p_2^a, p^w; \phi_2) \]

The water equilibrium remains as in [ ].

### 2.6.3 Partial Specialization

Partial specialization refers to the case where \( \beta_1 \) is sufficiently high such that only region 1 produces the agricultural good, however both regions may continue to produce the manufacturing good, i.e. there may not be complete concentration of the population in one region. As in the supplemental equilibrium the agricultural price relationship is given by, \( p_2^a = \tau p_1^a \). Given that no agriculture is produced, the region 2 agricultural rent is 0 and no agricultural water is used. The agricultural goods equilibrium is given by,

\[ (1-\lambda) N f_1(w_1^a(p_1^a, p^w; \beta_1); \beta_1) = \lambda N a_1(I_1, p_1^a, p^w; \phi_1) + (1-\lambda) N a_2(I_2, p_2^a, p^w; \phi_2) \]
While the water use equilibrium is given by,

\begin{equation}
W = \lambda N w_1^u(I_1, p_1^a, p^w; \phi_1) + (1 - \lambda) N w_2^u(I_2, p_2^a, p^w; \phi_2) + (1 - \lambda) N w_1^a(p_1^a, p^w; \beta_1),
\end{equation}

where the region 2 agricultural water use is omitted from \[.\]

### 2.6.4 Spatial Equilibrium

In the long run households locate where they can achieve the highest utility. Therefore, for a spatial equilibrium to occur, utility must be equal across regions, yielding,

\begin{equation}
V_1(I_1, p_1^a, p^w; \phi_1) = V_2(I_2, p_2^a, p^w; \phi_2)
\end{equation}

\[closes the model by defining the equilibrium population share \( \lambda \) as a function of model parameters.

### 3 An Example with Specific Functional Forms

This section considers a specific case of the model where the agricultural production function is chosen to allow for closed form solutions of the model. The numerical simulation will use a more flexible functional form for agricultural production too allow for more realistic calibration. Assume that \( W = N \) and that \( L_s = L \). This implies that there is one unit of water per person and that the length separating the regions is equivalent to the size of the regions themselves. Productivity in the manufacturing sector, is assumed to be given by,

\begin{equation}
A_1(\lambda; \delta) = A(1 + \lambda)^{\delta}
\end{equation}

\begin{equation}
A_2((1 - \lambda); \delta) = A(1 + (1 - \lambda))^{\delta},
\end{equation}

where \( A \) is a positive constant. \[ and \[ then indicate the regional household wage. The commuting costs are \( t(x_i) = tx_i \), which implies that the rental transfer is given by,

\[ \bar{r} = r_1^a + r_2^a + \frac{tN}{2}(\lambda^2 + (1 - \lambda)^2). \]
Household preferences are assumed to be Cobb-Douglas with the utility function given by,

\begin{equation}
U_i(m_i, a_i, w_i^u; \phi_i) = \phi_i m_i^\eta a_i^\mu (w_i^u)^\gamma.
\end{equation}

Household demand functions are,

\begin{equation}
m_i = \eta I_i, \quad a_i = \mu \frac{I_i}{p_i^a}, \quad w_i^u = \gamma \frac{I_i}{p_i^w}.
\end{equation}

The indirect utility function is then,

\begin{equation}
V_i(I_i, p_i^a, p_i^w; \phi_i) = \phi_i \eta^\eta \mu^\mu \gamma^\gamma I_i \left(\frac{p_i^a}{p_i^w}\right)^{\mu} \left(\frac{p_i^w}{p_i^w}\right)^\gamma.
\end{equation}

The production function per unit of land is assumed to be,

\begin{equation}
f(w_i^a; \beta_i) = 2\beta_i \sqrt{w_i^a}.
\end{equation}

This yields the following water demand and land rent functions,

\begin{equation}
w_i^a = \left(\frac{\beta_i p_i^a}{p_i^w}\right)^2, \quad r_i^a = \left(\frac{\beta_i p_i^a}{p_i^w}\right)^2.
\end{equation}

The infrastructure tax, \( f \), can be written as,

\begin{equation}
f = (1 + \lambda N I_1 + (1 - \lambda) N r_i^a) - p_w.
\end{equation}

Finally, the regional net incomes are,

\begin{align}
I_1 &= A(1 + \lambda)^\delta + r_i^a + \frac{t N}{2}(2\lambda^2 - 4\lambda + 1) + p^w - (1 + \frac{\lambda N I_1 + (1 - \lambda) N r_i^a}{p^w}), \\
I_2 &= A(1 + (1 - \lambda))^\delta + r_i^a + t N(\lambda^2 - \frac{1}{2}) + p^w - (1 + \frac{\lambda N I_1 + (1 - \lambda) N r_i^a}{p^w}).
\end{align}

It is straightforward to verify that there is an equilibrium where the population is evenly split when \( \phi_1 = \phi_2, \beta_1 = \beta_2, \tau = 1 \) and \( \delta = 0. \)
3.1 Autarky

Under autarky, using $[]$ and $[]$ the agricultural goods and water equilibrium imply the following,

\begin{equation}
\begin{align*}
\frac{d}{a_1} &= \frac{\mu}{2} \frac{\lambda}{1-\lambda} I_1, \\
\frac{d}{a_2} &= \frac{\mu}{2} \frac{1-\lambda}{\lambda} I_2, \\
p^w &= (\gamma + \frac{\mu}{2})(\lambda I_1 + (1-\lambda)I_2)
\end{align*}
\end{equation}

Finally, the spatial equilibrium is given by,

\begin{equation}
\Phi I_1 = I_2,
\end{equation}

where,

$$
\Phi = \frac{\phi_1}{\phi_2} \left( \frac{p_{b_1}^o}{p_{b_2}^o} \right)^\mu \cdot \left( \frac{p_{a_1}^o}{p_{a_2}^o} \right)^\mu = \left( \frac{\phi_1}{\phi_2} \right) \left( \frac{\lambda \beta_1}{(1-\lambda)\beta_2} \right)^2 \left( \frac{\mu}{\gamma+\mu} \right).
$$

Combining $[]$ and the definition of $I_1$, after manipulation yields,

\begin{equation}
I_1^A(\lambda) = \frac{(A(1+\lambda)^\delta + \frac{tN}{2}(2\lambda^2 - 4\lambda + 1) - 1) - \lambda}{1 - (\gamma + \frac{\mu}{2})(\lambda + \Phi(1-\lambda)) - \Phi \frac{\mu}{2}} \left( \frac{\lambda}{\lambda+\Phi(1-\lambda)} \right)
\end{equation}

where the $A$ subscript denotes the autarkic region 1 income. Substituting $[]$ into $[]-[]$ allows for the remaining endogenous variables to be written as functions of $\lambda$. Combining with the spatial equilibrium condition yields an implicit solution for $\lambda$,

\begin{equation}
\left( A(1+\lambda)^\delta + \frac{tN}{2}(2\lambda^2 - 4\lambda + 1) - 1 - \frac{\lambda}{\lambda+\Phi(1-\lambda)} \right) = \left( \frac{A(1+\lambda)^\delta + tN(\lambda^2 - \frac{1}{2}) - 1 - \frac{\lambda}{\lambda+\Phi(1-\lambda)}}{\Phi - (\gamma + \frac{\mu}{2})(\lambda + \Phi(1-\lambda)) - \frac{\mu}{2}} \right)
\end{equation}

Figures $[]-[]$ trace out the effects of changes in $\beta_1$, $\phi_1$ and $\delta$ from the symmetric equilibrium.

An increase agricultural productivity reduces both the price of both water and the agricultural good in region 1. This leads to a small increase in the region 1 population. Meanwhile the only effect an increase in the level of natural amenities, $\phi_1$, is an increase in the price of water and a slight migration towards region 1. Similarly, an increase in manufacturing productivity, $\delta$ leads to an increase in $p^w$. There is a slight increase in the utility level among both regions when $\delta$ increases however $\lambda$ remains at $1/2$. This follows from the fact that when $\lambda$ is close to
1/2 gross wages are roughly the same in each region.
Figure 5: Effect of $\beta_1$, $\phi_1$ and $\delta$ on equilibrium population share and prices under autarky. Note: Baseline parameters $\{N=1000, A=1000, t=.1, W=1000, \phi_1 = 1, \phi_2 = 1, \beta_1 = 1, \beta_2 = 1, \delta = 0, \tau = 1, \mu = .2, \gamma = .05\}$. 

- $\beta_1=1, \beta_1=1.3$,
- $\phi_1=1, \phi_1=1.02$,
- $\delta=0, \delta=.075$ 

$U_1 - U_2$
3.2 Incomplete Specialization

Recall that under supplemental trade the agricultural price relationship is given by, \( p_2^a = \tau p_1^a \). The equilibrium conditions then yields the following relationships

\[
(48) \quad r_1^a = B r_2^a, \quad r_2^a = \frac{\mu}{\tau} \left( \frac{I_1}{\tau} + \frac{(1-\lambda)I_2}{\tau} \right), \quad p^w = \left( \gamma + \frac{\mu}{2} \right) \lambda I_1 + \left( \gamma + \frac{\mu}{2\tau} \right)(1-\lambda)I_2
\]

where

\[
B \equiv \left( \frac{\beta_1}{\tau \beta_2} \right)^2
\]

The spatial equilibrium is then given by,

\[
(49) \quad TI_1 = I_2
\]

where

\[
T \equiv \frac{\phi_1 \tau^\mu}{\phi_2}
\]

Solving for \( I_1 \) yields,

\[
(50) \quad I_1^S = \frac{[A(1 + \lambda)^\delta + \frac{tN}{2}(2\lambda^2 - 4\lambda + 1) - 1] - \frac{(B\lambda(1-\lambda)(\gamma + \frac{\mu}{2}) + \gamma \lambda^2 + B \frac{\mu}{2\tau}(1-\lambda)^2)}{\Lambda}}{1 - \frac{\Lambda}{(B(1-\lambda)+\frac{T}{\tau})} - \frac{\frac{\mu}{\tau}(1-\lambda)}{(B(1-\lambda)+\frac{T}{\tau})}}
\]

where

\[
\Lambda = (B(1-\lambda) + \frac{\lambda}{\tau})(\lambda(\gamma + \frac{\mu}{2}) + (1-\lambda)\frac{T}{\tau}(\gamma + \frac{\mu}{2\tau})
\]

As in the autarkic case, equation \([\text{ ]}\) can be used to derive the implicit spatial equilibrium condition, given by,

\[
(51) \quad \frac{[A(1 + \lambda)^\delta + \frac{tN}{2}(2\lambda^2 - 4\lambda + 1) - 1] - \frac{(B\lambda(1-\lambda)(\gamma + \frac{\mu}{2}) + \gamma \lambda^2 + B \frac{\mu}{2\tau}(1-\lambda)^2)}{\Lambda}}{1 - \frac{\Lambda}{(B(1-\lambda)+\frac{T}{\tau})} - \frac{\frac{\mu}{\tau}(1-\lambda)}{(B(1-\lambda)+\frac{T}{\tau})}} = T - \frac{\Lambda}{(B(1-\lambda)+\frac{T}{\tau})} - \frac{\frac{\mu}{\tau}(1-\lambda)}{(B(1-\lambda)+\frac{T}{\tau})}
\]
Figures [-] trace out the equilibrium, for changes in $\beta_1$, $\phi_1$, $\tau$ and $\delta$. Under the supplemental trade regime, an increase in the agricultural productivity generates more variation than under autarky. There are significant shifts downward in all prices except for the region 1 agricultural rent, which remains roughly the same. The relative rise in rents between region 1 and 2, and the fall in agricultural prices in both regions, leads to a shift of households from region 1 to 2, and an increase in utility. Meanwhile, an increase in natural amenities in region 1, raises all prices and shift households towards region 1. As would be expected an increase in transport costs, increases $\lambda$ and decreases overall utility. Agricultural rents and prices fall in region 1 while they rise in region 2 which is the cumulative effect of the increase in tau and the price of water. Finally, improvements in manufacturing productivity, $\delta$, increases all prices and shift the majority of household to region 1 at a significant utility gain. This is a consequence of the asymmetry of the population in the initial equilibrium. When the population share differential is high, increases in $\delta$ benefit the more populous region to a greater extent.
— $\beta_1=1.3$, $\beta_1=1.7$ — $U_1$, $U_2$. — $\phi_1=1$, $\phi_1=1.02$

Figure 6: Effect of $\beta_1$, $\phi_1$ and $\delta$ on equilibrium population share and prices under supplemental trade. Note: Baseline parameters $\{N=1000, A=1000, t=.1, W=1000, \phi_1=1, \phi_2=1, \beta_1=1, \beta_2=1, \delta=0, \tau=1, \mu=\gamma=0.05\}$
Figure 7: Effect of $\beta_1$, $\phi_1$ and $\delta$ on equilibrium population share and prices under supplemental trade. Note: Baseline parameters \{N=1000, A=1000, $t=.1$, W=1000, $\phi_1=1$, $\phi_2=1$, $\beta_1=1$, $\beta_2=1$, $\delta=0$, $\tau=.075$\}.
3.3 Agricultural Specialization

In the case that only region 1 produces agriculture there is no agricultural rent in region 2. The agricultural and water equilibrium are then given by,

\[ r_1^a = \frac{\mu}{2} \left( \frac{\lambda}{1 - \lambda} I_1 + \frac{I_2}{\tau} \right), \quad p^w = (\gamma + \frac{\mu}{2}) \lambda I_1 + (\gamma + \frac{\mu}{2\tau}) (1 - \lambda) I_2 \]

The spatial equilibrium again is given by,

\[ TI_1 = I_2, \]

where \( T \) is defined as in [ ]. Following a similar algorithm as above, region 1 income under specialization is given as,

\[ I_{11}^P = \frac{A(1 + \lambda)^\delta + \frac{tN}{\tau}(2\lambda^2 - 4\lambda + 1) - 1 - \frac{(\gamma + \frac{\mu}{2}) \lambda + (1 - \lambda) \frac{\tau}{\Theta}}{1 - \Theta}}{1 - \Theta} \]

where the superscript \( P \) denotes partial specialization and

\[ \Theta \equiv (\lambda(\gamma + \frac{\mu}{2}) + (1 - \lambda)T(\gamma + \frac{\mu}{2\tau})) \]

The spatial equilibrium condition is then given by,

\[ \frac{A(1 + \lambda)^\delta + \frac{tN}{\tau}(2\lambda^2 - 4\lambda + 1) - 1 - \frac{(\gamma + \frac{\mu}{2}) \lambda + (1 - \lambda) \frac{\tau}{\Theta}}{1 - \Theta}}{1 - \Theta} = A(1 + (1 - \lambda))^\delta + \frac{tN(\lambda^2 - \frac{1}{2}) - 1 - \frac{(\gamma + \frac{\mu}{2}) \lambda + (1 - \lambda) \frac{\tau}{\Theta}}{1 - \Theta}}{T - \Theta - \frac{\tau}{2} \left( \frac{1 - \lambda}{1 - \lambda} + \frac{\tau}{\Theta} \right)} \]

Figures, [ ]-[] plot the the effects of \( \beta_1, \phi_1, \tau \) and \( \delta \) on the equilibrium.
Figure 8: Effect of $\beta_1$, $\phi_1$ and $\delta$ on equilibrium population share and prices under partial specialization. Note: Baseline parameters \{N=1000, A=1000, t=.1, W=1000, $\phi_1 = 1$, $\phi_2 = 1$, $\beta_1 = 1$, $\beta_2 = 1$, $\delta = 0$, $\tau = 1$, $\mu = .2$, $\gamma = .05$\}
Figure 9: Effect of $\beta_1$, $\phi_1$ and $\delta$ on equilibrium population share and prices under partial specialization. Note: Baseline parameters \{N=1000, A=1000, t=.1, W=1000, $\phi_1 = 1$, $\phi_2 = 1$, $\beta_1 = 1$, $\beta_2 = 1$, $\delta = 0$, $\tau = 1$, $\mu = .2$, $\gamma = .05$\}
Under specialization, a majority of households reside in region 2 allowing for region 1 to be primarily used for agricultural production, which now supplies all households in both regions. Notice that while an increase in productivity generates a substantial increase in utility there is no effect on migration. This follows from [], where the spatial equilibrium is independent of $\beta_1$. The rise in utility, is derived implicitly through the fall in agricultural prices. The effects of $\phi_1$ and $\tau$ are similar to that under supplemental trade. However, an increase in manufacturing productivity leads to concentration of all households in region 2. As the population share differential increases, an increase in $\delta$ intensifies migration towards the more populous region.

The next section describes the functional forms and calibration for the numerical exercise.

4 Functional Forms, Calibration and Policy Evaluations

Household utility is given by,

$U_i(a_i, m_i, w_i^u) = \phi_i m_i^{\eta_i} a_i^{\mu_i}(w_i^u)^{\gamma_i}$.

The share parameters are set at $\eta = .75$, $\mu = 0.2$ and $\gamma = 0.05$ so that the household share of net income devoted to water and agriculture is 25%. The agricultural production function takes the CES form,

$F_i(L_i^a, W_i^a; \beta_i) = \beta_i(\alpha(L_i^a)^{\rho} + (1 - \alpha)(W_i^a)^{\rho})^{1/\rho}, \ -\infty \leq \rho \leq 1$

where $\rho$ defines the elasticity of substitution, $\sigma$, between land and water, with $\sigma = \frac{1}{1-\rho}$. Consistent with empirical estimates (Luckman, et. al. (2014), Graveline and Merel (2014)), $\sigma$ is set at 0.2 which implies $\rho = -4$. $\alpha$ denotes the share of costs devoted to land and is set at 0.5. The model is calibrated to exhibit a number of stylized facts. Consistent with USGS estimates that roughly 1 acre foot of water is used per household in the state of California, the total population is set equal to the available water supply. The urban agglomeration parameter $\delta$ is set at 0.075 (Nijkamp and Verhoef(), Helsley and Sullivan()) while the threshold transport cost $\tau$ is set at 1.2 (Volpe, et. al. (2013)). The regional preference parameter $\phi_1 = 1.02$, while $\phi_2$ will be fixed at unity. The agricultural productivity parameter will be fixed at 1 for region 2 and vary between 1 and 2 for region 1, consistent with the USDA data on regional agricultural TFP. The commuting cost is set such that in the symmetric equilibrium, households who live at
Table 3: Parameter Values

<table>
<thead>
<tr>
<th>Benchmark</th>
<th>Model</th>
<th>Technology</th>
<th>Free Variables</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \tau = 1 )</td>
<td>( \tau = 1.2 )</td>
<td>( \eta = 0.75 )</td>
<td>( A = 1000 ),</td>
</tr>
<tr>
<td>( \phi_1 = 1 )</td>
<td>( \phi_1 = 1.02 )</td>
<td>( \mu = 0.2 )</td>
<td>( W = 1000 )</td>
</tr>
<tr>
<td>( \delta = 0 )</td>
<td>( \delta = 0.075 )</td>
<td>( \gamma = 0.05 )</td>
<td>( L_s = 300 )</td>
</tr>
<tr>
<td>( \rho = -4 )</td>
<td>( \alpha = 0.5 )</td>
<td>( \theta = 0.6 )</td>
<td></td>
</tr>
<tr>
<td>( N = 1000 )</td>
<td>( t = 0.1 )</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

the boundary of the city spend 5% of their gross income on transportation. Finally, the length of land separating the two regions is given by \( L_s = 0.3L \). Table [3] presents the parameters values.

The base case will be compared to a benchmark model where the only asymmetry is in the regional agricultural productivity, i.e. \( \delta = 0, \phi_1 = \phi_2 \). Additionally, we will consider the the social planners problem. In this case a social planner chooses the quantity of water and land to devote to agricultural production, the size of the city in each region, and the allocation of final goods to households in order to maximize utility. The problem is formally given as,

\[
\begin{align*}
\max_{\lambda, W_1^a, W_2^a, a_1, a_2, m_1, m_2} & \ U_1(m_1, a_1, w_1^u) \\
\text{s.t.} & \ U_1(m_1, a_1, w_1^u) = U_2(m_2, a_2, w_2^u), \\
& \ F_1(L_1^a, W_1^a; \beta_1) + F_2(L_2^a, W_2^a; \beta_2) - (\tau - 1)ES_1 - (\tau - 1)ES_2 \geq \lambda Na_1 + (1 - \lambda)Na_2, \\
& \ W \geq W_1^a + W_2^a + \lambda Nw_1^u + (1 - \lambda)Nw_2^u, \\
& \ \lambda N(A(1 + \lambda)^\delta + (1 - \lambda)N(1 + (1 - \lambda))^\delta \geq \lambda m_1 + (1 - \lambda)m_2 \\
& \quad + N(W + \frac{\lambda Nw_1^a + (1 - \lambda)Nw_2^a}{W}) + \frac{N}{2}(\lambda^2 N + (1 - \lambda)^2 N), \\
& \ ES_i \geq 0, \quad i = 1, 2.
\end{align*}
\]

where \( ES_i \) are excess supply functions for agricultural output in each region,

\[
ES_1 = F_1(L_1^a, W_1^a; \beta_1) - \lambda Na_1, \quad ES_2 = F_2(L_2^a, W_2^a; \beta_2) - (1 - \lambda)Na_2.
\]

Any excess supply that is exported uses the transport technology such that \((\tau - 1)ES_i\) is lost in
transit. Equation [6] is the manufacturing equilibrium. The left hand side is aggregate output, while the right hand side is the sum of household demand for manufacturing goods, the water distribution infrastructure and the household commuting infrastructure.

Finally, two policy evaluations will be conducted. The first, will examine the effects of subsidizing agricultural water by assuming the agricultural price is a constant share of the urban water price, $p_w^a = \theta p_w$. In this case the infrastructure tax is given by

$$f = \left(1 + L_s \left(\frac{\lambda N_w^1 + (1 - \lambda) N_w^2}{W}\right)\right) - p_w^r + (1 - \theta)p_w^r((1 - \lambda)w_1^q + \lambda w_2^q),$$

where the last term on the right is the household share of the additional revenue to cover the water subsidy. A second policy experiment will solve for region specific water prices, $p_1^w$ and $p_2^w$. It follows that the infrastructure tax becomes,

$$f = \left(1 + L_s \left(\frac{\lambda N_w^1 + (1 - \lambda) N_w^2}{W}\right)\right) - p_1^w \frac{(\lambda N_w^2 + (1 - \lambda) N_w^1)}{N} - p_2^w \frac{(1 - \lambda) N_w^2 + \lambda N_w^1}{N}. $$

The following section presents the results of the numerical simulation.

## 5 Numerical Simulation

For the autarkic case $\beta_1 = 1.3$ is chosen so that agricultural price ratio between region 2 and 1 is below the threshold transport cost to ensure that trade will not occur between regions, i.e. $\frac{p_2^a}{p_1^a} < \tau$. $\beta_1 = 1.7$ is chosen in the supplemental trade case so that trade occurs and both regions produce agriculture. Finally, in the case of specialization, a value of $\beta_1 = 2$ is chosen so that no agricultural production would occur in region 2. Tables [] and [] present the numerical results. The discussion will focus on three aspects: Relative prices, water allocation, and utility and migration.

### 5.1 Autarky

#### 5.1.1 Relative Prices

The relative price of agriculture between region 1 and 2 fall when natural amenities and increasing returns are introduced, in relation to the benchmark, while relative agricultural rents
rise. Agricultural water subsidies reduce this effect as a reduction in water costs and the high
degree of complementarity in factors drives up the demand for land from agriculture. Under
differential water pricing, there is a minor increase in the relative agricultural prices and the fall
in the land rents relative to the base case. Notice that regional water prices are not significantly
different, with the region 1 price less than 2.5% higher than region 2.

5.1.2 Water Allocation

Household water use remains roughly the same over all cases, however there is significant
variation in agricultural water use. The social planner uses water most intensively for region
1 agriculture and the least for region 2 over all scenarios, except in the case where agricultural
water is subsidized, with a ratio \( \frac{w_{a1}}{w_{a2}} = 1.248 \). When agricultural water is subsidized both regions
use more water than is optimal.

5.1.3 Utility and Migration

As would be expected, utility is lowest under the benchmark and highest under the social
planner. However, under the social planner less than half the population would reside in region
1 while under all other scenarios more than half reside in region 1, with the largest share located
in region 1 when water is subsidized.

5.2 Supplemental Trade

5.2.1 Relative Prices

Under supplemental trade relative rents between region 1 and 2 are highest under subside-
dized water at nearly 8:1 and lowest under differential water pricing at roughly 4:1. This follows
from the fact that when region 1, agricultural producers face a higher price for water, it reduces
the residual between revenue and water costs that can be captured by the agricultural rent.
Therefore, region 2 agricultural becomes relatively more competitive. In contrast, when water
is subsidized and common across both regions, agricultural rents in region 1 are bolstered by
the lands higher productivity as well the natural amenities, which increases the bid price for
land from households. With differential water pricing, the region 1 price is over 20% higher
than that of region 2.
5.2.2 Water Allocation

Household water use is significantly lower in both regions under subsidized water pricing and in region 1 under differential pricing. The subsidy on agricultural water increases the agricultural water demand and lowers the available share for households. However, under differential pricing, region 2 households consume nearly 30% more water than region 1 households. The social planner allocates the largest amount to region 1 relative to region 2 at \( \frac{w_1}{w_2} = 1.73 \). Under differential pricing the ratio is lowest with \( \frac{w_1}{w_2} = 1.14 \). As in the autarkic case, in all scenarios, other than subsidized water pricing, too little water is allocated to region 1 agriculture while too much is given to the less productive region. Under subsidized water, the water allotted to region 1 is roughly in line with the social planners allocation, however, the quantity used in region 2 is significantly above the social planners allotment.

5.2.3 Utility and Migration
Table 4: Numerical Results for Autarkic and Supplemental Equilibrium

<table>
<thead>
<tr>
<th></th>
<th>Benchmark</th>
<th>Base Case</th>
<th>Social Planner</th>
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<th>Regional Water Prices</th>
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Table 5: Numerical Results for Specialization Equilibrium

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5.3 Specialization

5.3.1 Relative Prices

Differential water pricing will not lead to partial specialization as the regional water price differential increases with $\beta_1$. The table then presents the supplemental trade results with $\beta_1 = 2$. Under differential water pricing the price ratio increases to $\frac{p_1}{p_2} = 1.65$ and the ratio of agricultural rents fall in comparison to when $\beta_1 = 1.7$. Among the other scenarios there is no significant change in relative equilibrium prices when water is subsidized versus the base case.

5.3.2 Water Allocation

The most notable effect is that the social planner utilizes water more intensively per unit of land than in all other scenarios. Due to the high prices of water in region 2, water is used the least intensively under differential pricing, with the quantities allotted per unit of land roughly the same in each region even though the productivity of region 1 is twice that of region 2.

5.3.3 Utility and Migration

Given that natural amenities are higher in region 1, the social planner continues to allocate roughly one third of the population to that region. Therefore, as noted above, water is used more intensively on the remaining land. Under the market scenarios too few households remain in region 1. When households have no preference for land as in the benchmark case, less than 20% locate in that region. Notice that under all scenarios in each regime, the base case performed the best in terms of utility relative to the social planner.

6 Future Research

For reasons of concision this paper has dealt with a very specific problem, namely, how will the uneven distribution of water, agricultural productivity and consumption amenities when transport costs are present, affect the size of the cities and agricultural production in each region. However, there are a number of other factors that play an important role in interbasin water transfers. One is the energy use needed to pump water through the network, particularly uphill over mountain ranges. The model could be adapted to take into account topographical irregularities, which would vary the marginal and fixed costs of distribution over space. In
addition, one could consider the possibility of electricity generation from the water flow in order to measure net energy use.

A timely extension would be to add the possibility of water desalination into the model. This could be done by introducing a water production technology that can add to the existing supply. Crucial questions include the scale and location of water production. Explicit dynamics could be introduced to solve for the optimal time to introduce the water desalination technology. In addition, variability in seasonal or annual water supply could be integrated.

There are environmental and ecological concerns related to interbasin water transfers, that may limit the extent to which they can be carried out. Integrating these constraints, in addition to increasing the level of realism, can also highlight alternative conservation methods to stretch existing water resources in the absence of substantial water transfer options.

Finally, the model is well equipped to answer the extent that regions that are water scarce can benefit from imported goods that are water intensive to produce. In addition, as water resources in many regions are becoming increasingly scarce, it will be necessary to identify in what location is the water put to best use given the possibility of transporting it.

7 Conclusion

This paper has developed a spatial two-region general equilibrium trade model with water as a mobile factor of production and heterogeneity between regions in consumption amenities, agricultural productivity and initial endowments of water. The model was solved analytically for a special case. A numerical simulation was then done to allow for a comparison across various policy scenarios. The analysis suggests that when trade cannot occur, a greater share of the population lives in the more agricultural productive region. When the same region has the additional benefit of natural amenities, the effect is compounded. When trade is possible, migration tends toward the less productive region, however, this effect is dampened if the more productive region has a higher level of natural amenities. In addition, it is that economies of scale play a significant role in migration patterns if the the population share differential between the two regions is sufficiently high. The numerical analysis showed that subsidizing agricultural water led to insufficient water being allocated to households and too much water was used by the less productive region. In contrast, under regional water prices, insufficient water was allocated to households and agriculture in the more productive region. A common price, provided the
highest utility among the market options analyzed.

8 Data Sources


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