Cultural workers and the character of cities

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Keywords:

Regional Migration, Spatial Distributions of Regional Economic Activity,

Housing Supply and Markets, Cultural Economics

JEL classification: R23, R31, R12, Z1

November 15, 2017

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I like to know, although I've never done it, I like to know that if at two o'clock in the morning I get a sudden urge for duck won-ton soup, that I can go downstairs, find a taxicab, go to Chinatown, get it and come back home. This is important to me.

Woody Allen on the virtues of New York City
Rolling Stone, April 1987

Abstract

This paper develops a two-region spatial model with heterogeneous agents in order to explore the tradeoffs between the market size in aggregate demand and market crowding effects in housing costs that cultural producers face in cities that foster industries with agglomeration economies. When all types of workers supply labor inelastically, we find that concentration is always a stable equilibrium and define the conditions for both a partially segregated and partially integrated equilibrium to be stable. An extension is considered where cultural producers can divide their time between working in the arts or in a constant returns production sector. The model and its extension are calibrated to US data. It is shown that some degree of dispersion is preferred to full agglomeration though the utility differential is decreasing in the housing supply of the agglomerated region and increasing in the elasticity of substitution between varieties of cultural goods.

1 Introduction

Cities often pride themselves on the scale and variety of their local cultural sector. Broadway in New York City, the West End in London, or the Sunset Strip in Los Angeles are emblems of a city's cultural arena and indicators of urban vitality. As cities grow, it is important to understand the role that the cultural sector and its producers play in a city's expansion. It has been well documented in the urban economics literature that the share of the population living in cities has been steadily rising over the last century. However, such growth has not been uniform across all cities, as certain metropolitan populations have become larger (New York, San Francisco Bay Area, Los Angeles) while others have dwindled (Detroit, Cleveland, St. Louis). With these shifting populations, so have the economic fortunes of the cities involved. Larger cities often provide a great variety of urban amenities, which, in turn, attract new residents. However, when there are large income differentials within a metropolitan area and a limited housing supply, high income households bid up rents beyond the reach of less affluent households. Therefore, thriving cities often struggle with providing affordable housing for lower income workers. Many such cities are finding much of the urban core primarily occupied by high-income workers while the peripheral cities are increasingly being settled by migrating workers looking for cheaper housing. This is particularly important with regards to cultural workers, many of whom are well trained and highly educated but often receive a lower income than their equally well-educated counterparts employed in other industries (Alper *et al.* (1996)).

Cultural producers that provide urban amenities are often caught in the divide between high and low-income residents in cities. One the one hand, locating in an affluent city where the demand for urban amenities is high provides a large income source for cultural producers. As casual empiricism suggests, high-income cities offer a greater variety of cultural goods and are able to accommodate more rarefied tastes (record stores devoted solely to jazz or classical music, specialty restaurants, experimental theater). On the other hand, when residing in such cities, cultural producers must also compete with high income workers for more expensive housing. In contrast, peripheral cities offer a lower income base from which cultural producers can draw from for their income, but are compensated with cheaper local rents. There is often a concern in cities where rents are rising rapidly about what will happen to the cultural makeup of the city as artists are unable to afford to live and work in those locations. In addition, a number of less prosperous cities have proven to be a haven for the cultural class: Asheville, NC; Tucson, AZ; and Athens, GA, for instance. Interestingly, Detroit's recent financial distress has been a boon to both local and relocating artists who have taken advantage of the low cost of housing from the spate of foreclosures over the years (see Ewing and Grady (2012)). To some extent, this is not surprising. Cities with a declining population retain a stock of both public and private infrastructure left behind by departing residents, which can be relatively quickly and inexpensively reconfigured for a new use. For example, popular cultural establishments in high-rent cities are often forced to close due to the high rental costs but resurface in lower-rent neighborhoods or cities nearby.

In this paper, we avoid normative statements regarding the quality of cultural work and make no direct comparison between, say, the value of fine art as compared to light entertainment (for a theoretical analysis of high and low art see Cowen and Tabarrok (2000)). This allows for a particularly broad view of the ingredients of a specific city's cultural makeup and can include, local restaurants and museums, music and theater venues, book and record stores. The primary focus is that households have some interaction in the city, through the consumption of the cultural goods beyond simply housing and employment needs.

Our model modifies the footloose entrepreneur model developed by Ottaviano and Forslid (2003) to explore the effect of agglomeration economies on the cultural make-up of cities. Additionally, this research extends the literature on land use in New Economic Geography (NEG) models, pioneered by Helpman (1998) and extended by Tabuchi (1998), Murata and Thisse (2005) Pflüger and Südekeum (2007) and Tabuchi and Pflüger (2010). In the standard core-periphery model,(Krugman (1991), Fujita *et al.* (1999)) the location of households is determined by the relative real wages between regions, which, in turn, is determined by the number of cheaper local varieties of horizontally differentiated goods in relation to the number of more expensive imported varieties. Thus, the equilibrium hinges critically on the freeness of trade between regions and the relative cost of variety to households across regions. Here we offer an alternative specification. Households have Dixit-Stiglitz preferences over a variety of horizontally differentiated, local cultural goods that are produced by cultural workers and require a fixed cost for the "talent" of a cultural worker. However, the cultural good is assumed not to be tradable. The intuition is that many urban amenities are perishable, an "experience" , that cannot be equivalently replicated in another location. Thus two regions may have similar but not identical varieties of a good (e.g. a Broadway play in comparison to its nationally touring companion with a different cast). What we argue is that a major draw of a city is not the relative cost of the same number of varieties that different cities offer; rather the varieties available in one city which are simply not available in the other.

We consider three types of mobile workers: cultural producers, workers in a traditional industry, and workers in a modern industry, which is considered to have external economies of scale. Each city is endowed with a fixed housing supply as in Helpman (1998). Initially, we study the migration pattern when each type of worker supplies labor inelastically to their respective industry. Under this assumption, the concentration of workers is always a stable equilibrium. Additionally, we derive the conditions for partially segregated cities, where modern and traditional workers live in opposite cities and cultural producers are divided between both regions. Finally, a partially integrated equilibrium is found where a positive share of all three types of workers are located in a single region.

There has been considerable research on artists labor markets which largely debunks the vision of the starving artist (Throsby (1992), Alper and Wassal (2006)). This is due to the fact that artists tend to be relatively well educated and have access to more lucrative outside options, which allows them to increase their earnings . Therefore, "moonlighting" in other industries provides cultural workers a more reliable income stream (See Alper and Wassal (2000)) . To account for this, our model is extended to allow for cultural producers to split their time between cultural production and production of the traditional good. We find that when the elasticity of substitution between varieties of cultural goods is sufficiently high, there is a stable equilibrium in which all traditional and modern workers concentrate in one region, while a portion of the cultural producers live separately in the opposite region.

The remainder of the paper is presented as follows. Section 2 develops the model and considers the various equilibria. Section 3 calibrates the model and presents the numerical results. Section 4 extends the model to allow for labor choice by cultural producers. Section 5 calibrates the adapted model and provides the results. Finally, section 6 concludes.

2 The Model

Consider two cities, indexed by $i = 1, 2$, which together have a total population equal to 3. Each household supplies labor inelastically to firms in one of three industries: modern, traditional, and cultural, which are indexed by $j = m, t, c$. Note that the terms *cities* and *regions*, as well as *cultural producers* and *artists* are used interchangeably in this paper. Workers in industry j located in region i receive the wage w_{ji}. Each industry has an exogenous total supply of labor, $\lambda_j = 1$, that is divided endogenously between both cities such that $\lambda_j = \lambda_{j1} + \lambda_{j2}$, where λ_{ji} is the endogenous share of a type j worker in region i. There is a fixed supply of homogeneous housing in each city, H_i, which carries the rental price r_i. Households have preferences over housing, H_{ji}, the numeraire traditional good, T_{ji} and the modern good, M_{ji}, which sells for the price p_m. Both the modern and traditional goods are assumed to be freely traded across regions. In addition, households have preferences over an aggregate of locally produced, horizontally differentiated cultural goods, C_{ji}, which carries the local price $p_{ci}(s)$ for each variety, s of the good, $c_{ji}(s)$. The utility function for a worker in industry j residing in region i is given by,

$$U_{ji} = T_{ji}^{\alpha} M_{ji}^{\beta} H_{ji}^{\gamma} C_{ji}^{\delta}, \quad \alpha + \beta + \gamma + \delta = 1, \tag{1}$$

$$C_{ji} = \left(\int_{s \in K} c_{ji}(s)^{(\sigma-1)/\sigma} ds \right)^{\sigma/(\sigma-1)}, \tag{2}$$

where, α, β , γ and δ are parameters denoting the share of income devoted to consumption of the traditional good, modern good, housing and cultural goods, respectively. The total mass of varieties is K, with the number of varieties in each region denoted k_i. The parameter $\sigma > 1$ represents the elasticity of substitution between each variety of the cultural good.

Utility maximization yields the following demand functions

$$T_{ji} = \alpha w_{ji}, \quad M_{ji} = \beta \frac{w_{ji}}{p_m}, \quad H_{ji} = \gamma \frac{w_{ji}}{r_i}, \quad C_{ji} = \delta \frac{w_{ji}}{P_i}, \tag{3}$$

where

$$P_{ci} = \left(\int_1^{k_i} p_{ci}(s)^{1-\sigma} \, ds \right)^{1/(1-\sigma)} \tag{4}$$

is the CES price index for cultural goods in region i. The demand for each variety is then

$$c_{ji}(s) = \delta \frac{w_{ji}}{p_{ci}(s)^{\sigma} P_{ci}^{1-\sigma}}. \tag{5}$$

Finally, the indirect utility function for a type j worker in region i is given by

$$V_i = \eta \frac{w_{ji}}{P_i}, \tag{6}$$

with

$$P_i \equiv p_m^{\beta} r_i^{\gamma} P_{ci}^{\delta}, \quad \eta \equiv \alpha^{\alpha} \beta^{\beta} \gamma^{\gamma} \delta^{\delta}, \tag{7}$$

where P_i represents the regional price index and η is a cluster of parameters.

2.1 Traditional and Modern Sector

Both the traditional and modern production sectors are perfectly competitive with linear production functions. Workers in the traditional sector produce one unit of output per unit of labor. Given that the traditional good is the numeraire and is freely traded we then have $p_t = w_{t1} = w_{t2} = 1$. The modern sector is comprised of a continuum of small firms who also produce output solely using labor with the perceived marginal product of labor A_{mi}, which firms take as given. However, there are Marshallian externalities generated from the share of modern workers in a city with $A_{mi} = A_m(1 + \lambda_{mi})^{\epsilon}$, where

A_m denotes the marginal product of an isolated worker and ϵ is a measure of the degree of scale economies in the industry. The wage of modern workers, $w_{mi} = p_m A_{mi}$, is then the value of their average product rather than their marginal product.

2.2 Cultural Sector

Each cultural producer uses B units of the numeraire good per unit of output in addition to a fixed cost of one unit of their labor. Total cost for producing a variety s in region i is then

$$TC_i(s) = Bc_i(s) + w_{ci}. \tag{8}$$

Equating the supply and demand for labor of cultural producers yields the number of varieties of cultural goods in each region

$$k_i = \lambda_{ci}. \tag{9}$$

The profit for a typical firm producing variety s in region i is given by,

$$\pi_i(s) = (p_{ci}(s) - B)c_i(s) - w_{ci}. \tag{10}$$

Under the Chamberlinian large group assumption the producer of each variety acts as a monopolist, taking the prices of other varieties as given. The profit maximizing price is then a constant markup over marginal cost

$$p_{ci}(s) = B\frac{\sigma}{(\sigma - 1)}. \tag{11}$$

Note that the price of all varieties is the same both within and across cities. We can then write $p_{ci}(s) = p_c$ and $c_{ji}(s) = c_{ji}$.

Free entry of firms drives profits to zero. Therefore the quantity of output of each variety is chosen such that any net revenue covers the fixed labor costs for cultural producers.

$$c_{ji} = \frac{(\sigma - 1)}{B} w_{ci}. \tag{12}$$

Finally, from (4) the CES price-index in each city can be written as

$$P_{ci} = B\frac{\sigma}{(\sigma-1)}\lambda_{ci}^{1/(1-\sigma)}. \tag{13}$$

Note that the price index is declining in the number of cultural producers in each region and thus in the size of the local cultural labor force.

2.3 Temporary Equilibrium

It is assumed that in the short run the share of each type of worker in either region is fixed. The market clearing conditions are then given by

$$p_m(\lambda_{m1}A_{m1} + \lambda_{m2}A_{m2}) = \beta\Big(p_m(\lambda_{m1}Am_1 + \lambda_{m2}A_{m2}) + 1 + \lambda_{c1}w_{c1} + \lambda_{c2}w_{c2}\Big), \tag{14}$$

$$\sigma\lambda_{ci}w_{ci} = \delta\Big(p_mA_{mi}\lambda_{mi} + \lambda_{ti} + \lambda_{ci}w_{ci}\Big), \tag{15}$$

$$r_iH_i = \gamma\Big(p_mA_{mi}\lambda_{mi} + \lambda_{ti} + \lambda_{ci}w_{ci}\Big), \qquad i = 1,2. \tag{16}$$

Solving for p_m, w_{ci} and r_i yields the short run prices

$$p_m = \frac{\sigma\beta}{\sigma(1-\beta)-\delta}\left(\frac{1}{\lambda_{m1}A_{m1} + \lambda_{m2}A_{m2}}\right), \tag{17}$$

$$w_{ci} = \frac{\delta}{\sigma-\delta}\left(\frac{\sigma\beta}{\sigma(1-\beta)-\delta}\frac{\lambda_{mi}A_{mi}}{\lambda_{m1}A_{m1} + \lambda_{m2}A_{m2}} + \lambda_{ti}\right)\frac{1}{\lambda_{ci}}, \tag{18}$$

$$r_i = \frac{\gamma\sigma}{\sigma-\delta}\left(\frac{\sigma\beta}{\sigma(1-\beta)-\delta}\frac{\lambda_{mi}A_{mi}}{\lambda_{m1}A_{m1} + \lambda_{m2}A_{m2}} + \lambda_{ti}\right)\frac{1}{H_i}. \tag{19}$$

Note that the price of the manufacturing good, p_m, is highest under even dispersion when $\lambda_{m1} = 1/2$ and lowest under agglomeration when $\lambda_{m1} = 1$ and $\lambda_{m1} = 0$. Furthermore, the values of the average product of the modern good, which is simply the modern workers wage, $w_mi = p_mA_{mi}$, are equal at $\lambda_{m1} = 1$, $\lambda_{m1} = 1/2$ and $\lambda_{m1} = 0$. Additionally, it can be shown that $\frac{\partial w_{mi}}{\lambda_{mi}}|_{\lambda_{mi}=1/2} > 0$ and $\frac{\partial w_{mi}}{\lambda_{mi}}|_{\lambda_{mi}=1} < 0$, so that the wage of modern workers in region i is maximized at some point $\lambda_{mi} \in (1/2, 1)$. For cultural workers, regional wages are decreasing in the local supply of artists, increasing in the number of traditional workers and increasing in modern workers up to the point that modern workers wages start to fall, as explained above. The effect of traditional and modern workers on local rents are similar to that of the wages of cultural workers. Additionally, rents are falling with the supply of housing. Note that the income of cultural workers is fully capitalized into rents, as λ_{ci} does not enter the short-run rent function.

2.4 Long-run Equilibrium

In the long-run, all workers locate in the region where they receive the higher utility. The adjustment process is assumed to take the form of the utility gap between regions for each type of worker

$$\dot{V}_j = \frac{d\lambda_{ji}}{dt} = V_{j1} - V_{j2}, \tag{20}$$

where t denotes time. There are stable concentrated equilibria at $\dot{V}_j \geq 0$ for $\lambda_{j1} = 1$ and $\dot{V}_j \leq 0$ for $\lambda_{j1} = 0$. Interior equilibria occur at $\dot{V}_j = 0$ for $\lambda_{j1} \in (0, 1)$. Interior equilibria are stable if the eigenvalues associated with the Jacobian of the adjustment process $\dot{V}_j$, $j \in \{t, m, c\}$ are negative or have negative real parts (See Tabuchi and Zeng (2004)).

2.5 Stable Equilibrium Configurations

We can now consider the different configurations of population across both cities. The possible distributions are listed below:

Integration: Each city contains all three types of workers

Concentration: All workers locate in a single city

Segregation: Each city has one type of worker and one city has two types

Partial Integration: Each city has two types of workers and one city has three types

Partial Segregation: Each city has two types of workers at most

Additionally, the following assumptions on the parameters are made:

Assumption 1: $\sigma > 1 + \delta$.

Assumption 2: $\frac{\sigma\beta}{\sigma(1-\beta)-\delta} > 1$

Assumption 3: $H_1 \geq H_2$.

Assumption 1 is the standard "no black hole condition", which ensures that all economic activity does not collapse to a single point. Assumption 2 ensures that the wage of modern workers is greater than that of traditional workers when concentrated in a single region. The final assumption is for narrative convenience, as the exposition will be focused on region 1. The model is symmetric in everything but housing supply; therefore reversing the inequality on Assumption 3 and focusing on region 2 will generate symmetric results.

2.5.1 Integration, Concentration and Segregation

Proposition 1. *Concentration is always a stable equilibrium while integration and segregation are never stable equilibria.*

Proof. First, consider the case of integration. Suppose there is an interior equilibrium such that $\dot{V}_c = 0$ and $\dot{V}_t = 0$, which implies that $P_1 = P_2$. Using this result, the equilibrium condition for modern workers is given by the sign of the utility gap

$$sign\,\dot{V}_m = sign\left((1+\lambda_{m1})^{\epsilon} - (1+(1-\lambda_{m1}))^{\epsilon}\right). \tag{21}$$

There is an interior spatial equilibrium at $\lambda_{m1} = 1/2$; however, the equilibrium is unstable as modern workers would deviate in order to increase their productivity and receive a higher wage by moving in greater numbers toward a single region.

Next, consider the case of concentration. Suppose all workers are located in region 1. Due to the Cobb-Douglas utility function, modern and traditional workers would never choose to deviate in order to ensure access to the cultural good. Therefore, we only need to consider whether cultural producers would choose to move. Suppose that a small measure of cultural producers, $d\lambda_{c1}$, is considering relocating to region 2. In this configuration, cultural producers in region 2 would derive their income selling the local varieties of cultural goods amongst each other. The market clearing condition in region

2 is then

$$\sigma w_{c2}\Big(d\lambda_{c1}\Big) = \delta w_{c2}\Big(d\lambda_{c1}\Big). \tag{22}$$

Given that $\sigma > \delta$, it must be that $d\lambda_{c1} = 0$ or $w_{c2} = 0$. Therefore, we can conclude that segregation is never a stable equilibrium and a concentrated equilibrium is always stable. □

Intuitively, when cultural and traditional workers are distributed across both regions, the cost of living is the same in each city. Therefore, modern workers are always better off chasing a higher wage, which is generated when modern workers move away from an even distribution. Conversely, when all workers are concentrated in a single region, a portion of cultural producers are motivated to move to the other city in order to take advantage of lower rents. However, the revenue from selling solely to other artists is insufficient to generate the revenue needed to cover the fixed costs of production. Therefore, cultural producers always return to the larger market.

2.5.2 Partial Segregation

We now consider the case of partial segregation. Assume that $\lambda_{m1} = 1$ and $\lambda_{t1} = 0$. As noted above, both modern and traditional workers demand a positive quantity of the cultural good, which ensures that cultural producers are distributed between both cities. The condition for the spatial equilibrium of cultural workers is given by

$$\begin{aligned} \dot{V}_c\Big|_{\substack{\lambda_{m1}=1\\ \lambda_{t1}=0}} &= \eta\left(\frac{w_{c1}}{P_1} - \frac{w_{c2}}{P_2}\right) \\ &= \kappa\left(\frac{\left(\frac{\sigma\beta}{\sigma(1-\beta)-\delta}\right)^{1-\gamma} H_1^{\gamma}}{\lambda_{c1}^{(\delta+1-\sigma)/(1-\sigma)}} - \frac{H_2^{\gamma}}{(1-\lambda_{c1})^{(\delta+1-\sigma)/(1-\sigma)}}\right) = 0, \end{aligned} \tag{23}$$

where

$$\kappa \equiv \frac{\eta}{(p_m)^{\beta}}\left(\frac{\sigma-\delta}{\sigma\gamma}\right)^{\gamma}\left(\frac{\sigma-1}{B\sigma}\right)^{\delta/1-\sigma}\left(\frac{\delta}{\sigma-\delta}\right)$$

is a bundle of common terms.

Solving (23) for λ_{c1} yields,

$$\lambda_{c1}^S = \frac{\left(\left(\frac{\sigma\beta}{\sigma(1-\beta)-\delta}\right)^{1-\gamma}(H_1)^\gamma\right)^\Phi}{(H_2^\gamma)^\Phi + \left(\left(\frac{\sigma\beta}{\sigma(1-\beta)-\delta}\right)^{1-\gamma}(H_1)^\gamma\right)^\Phi}, \tag{24}$$

$$\Phi \equiv \frac{\sigma - 1}{\sigma - 1 - \delta} > 0$$

where S is a mnemonic for partial segregation.

Eq. (24) has a clear intuitive explanation that focuses on the tradeoff cultural producers face between affordable housing and market demand for their products. The terms, $\left(\frac{\sigma\beta}{\sigma(1-\beta)-\delta}\right)^{1-\gamma}(H_1)^\gamma$, and, H_2^γ, are Cobb Douglas aggregations of the average income of non-cultural workers in each region (recalling that $w_t = 1$) and each region's housing stock weighted, respectively, by the expenditure share on non-housing consumption $1-\gamma$, and housing, γ. When γ is large, the supply of housing is weighted more heavily as a larger stock of housing reduces rents. When γ is relatively small, the average regional income of non-cultural workers plays a greater role in the distribution of cultural workers. Additionally, we have the following comparative statics:

$$\frac{\partial\lambda_{c1}^S}{\partial H_1} > 0, \quad \frac{\partial\lambda_{c1}^S}{\partial H_2} < 0, \quad \frac{\partial\lambda_{c1}^S}{\partial \sigma} < 0, \quad \frac{\partial\lambda_{c1}^S}{\partial \delta} > 0, \quad \frac{\partial\lambda_{c1}^S}{\partial \beta} > 0. \tag{25}$$

The signs in (25) show that the number of cultural producers in region 1 increases with the local supply of housing, H_1, which reduces local rents. While an increase in housing in region 2, H_2, reduces the supply of cultural workers as local rents become relatively more expensive. Note that the wage of modern workers, $\frac{\sigma\beta}{\sigma(1-\beta)-\delta}$, falls with σ and increases with δ and β. Therefore, an increase in σ has two effects. First, it reduces the monopoly power, and thus the wage of each cultural producer, as goods become more substitutable. Second, it reduces the income of modern workers. The cumulative effect is to reduce the number of cultural producers in region 1. An increase in δ has the opposite effect. It increases the share of income that all types devote to cultural goods but, in addition, it raises the wage of modern workers shifting more cultural producers to region 1. While an increase in β raises the wage of modern workers and thus the wages of region 1 cultural producers.

From (23) it is straightforward to show that the slope of $\dot{V}_c$ is negative as it crosses the equilibrium point provided that $\sigma > 1 + \delta/\gamma$. In order to ensure that (24) is a stable equilibrium, it must be verified that

$$\dot{V}_m\Big|_{\lambda_{c1}^S} \geq 0, \quad \dot{V}_t\Big|_{\lambda_{c1}^S} \leq 0. \tag{26}$$

The following set of inequalities provide the conditions for a stable equilibrium.

$$\dot{V}_t \leq 0 \iff \left(\frac{H_1}{H_2}\right)^{\psi} \leq \left(\frac{\sigma\beta}{\sigma(1-\beta)-\delta}\right) \tag{27}$$

$$\dot{V}_m \geq 0 \iff 2^{\epsilon}\left(\frac{H_1}{H_2}\right)^{\psi} \geq \left(\frac{\sigma\beta}{\sigma(1-\beta)-\delta}\right) \tag{28}$$

$$\psi \equiv \frac{\gamma(\sigma-1)}{\gamma(\sigma-1)-\delta} \tag{29}$$

The inequality in (27) ensures that the cost of living is lower in region 2 than region 1 for traditional workers, while (28) ensures that the productivity, and thus the wage, when all modern workers are concentrated in region 1 is sufficiently high to offset the higher cost of living. The bundle of parameters represented by ψ comes up repeatedly in the analysis, in particular, with respect to the relative housing stock between region 1 and 2. It is then useful to describe its components. ψ is positive if $\sigma > 1 + \delta/\gamma$ and if this holds, ψ is greater than 1. Additionally ψ is increasing in δ and decreasing in γ and σ. It follows that when H_1 is much larger than H_2 and σ is sufficiently low so that ψ is positive and large, the condition in (27) is less likely to hold. Therefore, partial segregation will tend to occur when each region has a similar supply of housing.

2.5.3 Partial Integration for Traditional Workers

This section considers an equilibrium configuration where modern workers are concentrated in a single city, while the other two types of workers are divided between cities. Again, assume $\lambda_{m1} = 1$. This equilibrium implies that $\dot{V}_c = 0$ and $\dot{V}_t = 0$. Solving for the population shares for cultural and traditional workers yields

$$\lambda_{c1}^I = \frac{H_1^\psi}{H_1^\psi + H_2^\psi} \quad \lambda_{t1}^I = \frac{H_2^\psi}{H_1^\psi + H_2^\psi}\left(\left(\frac{H_1}{H_2}\right)^\psi - \left(\frac{\sigma\beta}{\sigma(1-\beta)-\delta}\right)\right) \tag{30}$$

where ψ is defined as above. The superscript I is a mnemonic for integration. In order to verify whether the equilibrium is stable, the eigenvalues associated with the Jacobian matrix of $\dot{V}_c$ and $\dot{V}_t$, evaluated at the equilibrium values, $\lambda_{m1} = 1$, λ_{c1}^I and λ_{t1}^I, must be negative or have negative real parts. It can be shown that the trace of the Jacobian is negative, thus if the determinant of the Jacobian is positive both eigenvalues are negative. This condition holds provided

$$\sigma > 1 + \frac{\delta}{\gamma}. \tag{31}$$

The inequality in (31) suggests that a partially integrated equilibrium is more likely to be stable when the expenditure share on housing exceeds that of cultural goods and/or there is a greater degree of substitutability between varieties of the differentiated products. Note that

$$\lambda_{t1}^I > 0 \iff \left(\frac{H_1}{H_2}\right)^\psi > \left(\frac{\sigma\beta}{\sigma(1-\beta)-\delta}\right). \tag{32}$$

The condition in (32) is the opposite of that for a stable segregated equilibrium set out in (27). Intuitively, when the purchasing power of modern workers is not overwhelming and the housing supply in region 1 is sufficiently large to moderate rents, some measure of traditional workers will choose to locate in region 1. Additionally, with the partial integration of traditional workers, while the share of traditional and cultural workers in region 1 increases with H_1 from (30) and the properties of ψ, they do so at a decreasing rate as σ increases. Furthermore, the population shares for cultural and traditional workers in region 1 never reach unity, as the lower rents in region 2 always ensure some positive amount of economic activity. The results for partial segregation and partial integration are summarized below.

Proposition 2. *When $\sigma > 1 + \delta/\gamma$, partial segregation is a stable equilibrium if*

$$2^\epsilon \left(\frac{H_1}{H_2}\right)^\psi > \left(\frac{\sigma\beta}{\sigma(1-\beta)-\delta}\right) > \left(\frac{H_1}{H_2}\right)^\psi$$

while partial integration is a stable equilibrium if

$$\left(\frac{H_1}{H_2}\right)^{\psi} > \left(\frac{\sigma\beta}{\sigma(1-\beta)-\delta}\right)$$

.

2.5.4 Partial Integration for Modern Workers

Finally, we consider whether there is an equilibrium where all traditional workers are located in a single city while modern and cultural workers are divided between cities. Due to the nonlinearity in the wage of modern workers close form solutions are not available. Suppose that $\lambda_{t1} = 1$. The condition for a concentrated equilibrium for traditional workers is given by

$$\dot{V}_t \geq 0 \implies 1 \geq \frac{P_1}{P_2}, \tag{33}$$

which says that the price index in city 2 must be at least as high as that of city 1. The condition for an interior equilibrium for modern workers is given by

$$\dot{V}_m = 0 \implies \frac{(1+\lambda_{m1})^{\epsilon}}{(1+(1-\lambda_{m1}))^{\epsilon}} = \frac{P_1}{P_2}. \tag{34}$$

This says that the relative productivity of modern workers between city 1 and 2 is equal to the relative price index between city 1 and 2. Combining the two conditions in (33) and (34) implies that traditional workers will concentrate in city 1 if $\lambda_{m1} \leq 1/2$. The case that $\lambda_{m1} = 1/2$ can be ruled out as the system is overdetermined with both $P_1 = P_2$ and $w_{c1} = w_{c2}$, providing two equations to solve for a single unknown, λ_{c1}. We then consider $\lambda_{m1} < 1/2$. Appendix A.1 shows that the following conditions are necessary in order for modern workers to be divided between regions:

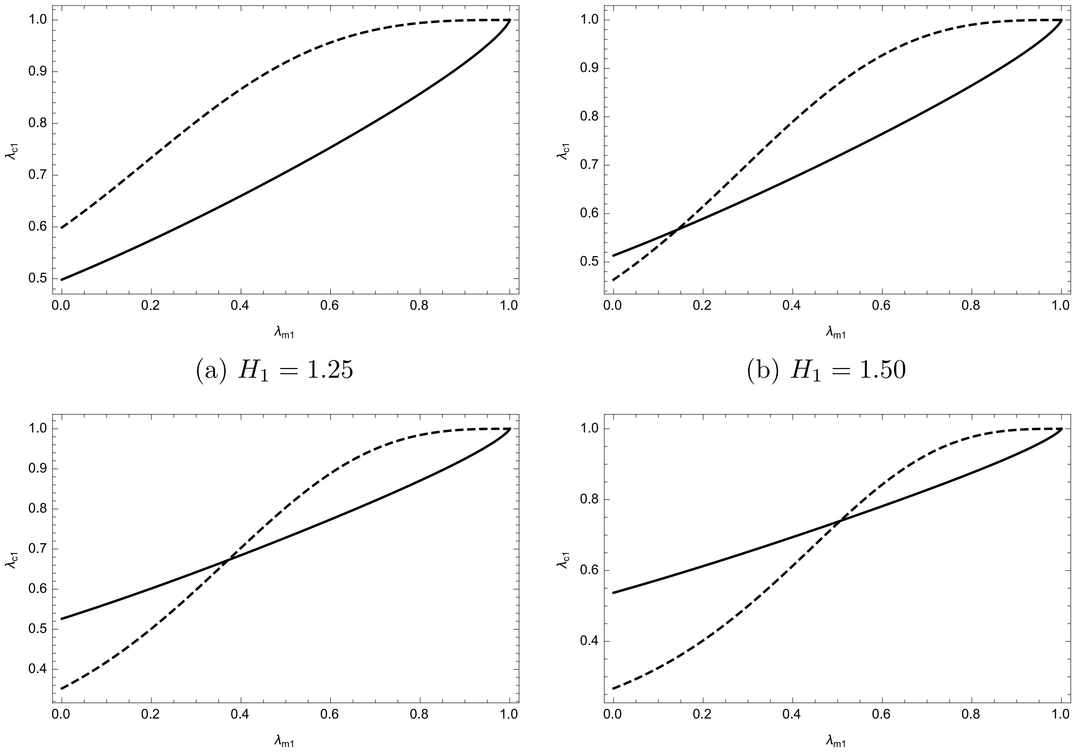
$$i) \quad \sigma > 1 + \delta/\gamma \tag{35}$$

$$ii) \quad \frac{\beta\sigma}{\sigma(1-\beta)-\delta}\left(\frac{H_1}{H_2}\right)^{\psi} > \left(2^{\epsilon}\right)^{\frac{\sigma-1-\delta}{\gamma(\sigma-1)-\delta}} \tag{36}$$

$$iii) \quad \frac{\beta\sigma}{\sigma(1-\beta)-\delta}\left(\left(\frac{H_1}{H_2}\right)^{\psi} - 1\right) < 2 \tag{37}$$

The condition in $i)$, which is consistent with the case of partial segregation, says that the elasticity of substitution must be sufficiently high. The condition in $ii)$ establishes that an interior equilibrium exists for modern workers. Note that this condition is more likely to hold if housing supply is significantly higher in city 1 relative to 2, and scale economies in the modern sector are low. Finally, the condition in $iii)$ ensures that $\lambda_{m1} < 1/2$. In contrast with $ii)$, the condition in $iii)$ is more likely to hold if housing supply in city 1 is similar to that in city 2. Conceptually, in order for traditional workers to concentrate, the cost of living in city 1 must be below that of city 2. For modern workers, when scale economies are low, there is less incentive to concentrate, thus pushing some workers towards the region with the larger supply of housing. If H_1 is sufficiently high, rents and thus the cost of living can remain relatively low in region 1, even with new migration. However, as H_1 increases so does the share of modern and cultural workers, which raises the cost of living in city 1 relative to city 2, breaking the condition in (33) and leading traditional workers to relocate.

Given that closed form solutions are not available, we cannot verify the stability of the equilibrium analytically. However, numerical simulations using parameter values consistent with $i) - iii)$ yield negative eigenvalues for the Jacobian matrix suggesting that the equilibrium is stable. Figure 1 provides a graphical example. In Figure (1a) the housing supply in region 1 is 25% greater than that of region 2 and the only equilibrium is concentration of all workers in city 1. Figure (1b) shows a stable interior equilibrium with roughly 15% of modern workers and 55% of cultural workers in city 1. When the supply of housing in region 1 is 75% larger than that of region 2, there is an equilibrium with nearly 40% of modern workers and 65% of cultural workers. Once H_1 increases to twice that of H_2 as in Figure (1d), the majority of all workers are located in city 1 therefore condition *iii*) is violated and the equilibrium in the interior is unstable.



(a) $H_1 = 1.25$ (b) $H_1 = 1.50$

(c) $H_1 = 1.75$ (d) $H_1 = 2$

Figure 1: Partial integration of modern workers with varying housing supply in region 1. Note: $\sigma = 2$, $\delta = .1$, $\beta = .5$, $\gamma = .25$, $\epsilon = .21$, $H_2 = 1$, $\lambda_{t1} = 1$.

3 Utility Comparison

$\beta = .5$	
$\gamma = .3$	(BLS, 2015)
$\delta = .1$	(BLS, 2015), (USDA ERS, 2016)
$\sigma = \{1.5, 2\}$	(Richards and Mancino, 2011)
$\epsilon = \{0.10, 0.21\}$	(Ciccone and Hall, 1996)
$H_1 = \{1, 1.20\}$	
$H_2 = 1$	

Table 1: Parameter Values

The preceding sections show that a concentrated equilibria is always stable and defined the conditions for the stability of the (partially) interior equilibria. This section considers numerically whether the concentrated or interior equilibria are preferred by each type of worker under reasonable parameter values. Table 1 provides the parameter values used in the calculations. To calibrate the Cobb-Douglas income expenditure shares, γ is set at 0.30, consistent with a data from the BLS (2002). δ is set at 0.10 which adds together estimates of the share of expenditure spent on entertainment (BLS 2015) and eating out (USDA ERS 2016) which are both near 5% of disposable income. In order to choose appropriate values for the parameters, it is necessary to take into account the conditions for the interior equilibria laid out in Assumptions 1-3, Proposition 2 and (33)-(35). Choosing $\beta = 0.50$ ensures that Assumption 2 holds. Given that relative housing supply is a key component, H_2 is fixed at 1 and H_1 is varied between 1 and 1.2. To calibrate the elasticity of substitution, σ, we use estimates on the elasticity of substitution between restaurant types (Richards and Mancino, 2011). The estimates vary between 1.2 and 3.3, and we consider values of 1.5 and 2. Finally, the value of ϵ is drawn from empirical estimates on agglomeration economies (Ciccone and Hall, 1996). We consider high and low values in order to ensure that the conditions on the parameters, as laid out in the preceding discussion, hold.

	$\sigma = 1.5$	$\sigma = 2$
Partial Segregation ($H_1 = 1$, $\epsilon = .21$)		
λ_{c1}	0.53	0.52
λ_{m1}	1.00	1.00
λ_{t1}	0.00	0.00
Partial Integration (Traditional) ($H_1 = 1.2$, $\epsilon = .21$)		
λ_{c1}	0.63	0.57
λ_{m1}	1.00	1.00
λ_{t1}	0.21	0.09
Partial Integration (Modern) ($H_1 = 1.2$, $\epsilon = .1$)		
λ_{c1}	0.55	0.51
λ_{m1}	0.15	0.05
λ_{t1}	1.00	1.00

Table 2: Population Shares. Note: $H_2 = 1$, $\beta = 0.5$, $\delta = .1$, $\gamma = .3$

Table 2 gives the population distribution under the different interior equilibria. Clearly, in each case cultural producers are divided. Though, in the case of partial integration where the bulk of the total population, and thus total income, is in region 1, the larger share of cultural producers also reside in region 1. Additionally, we see that an increase in σ reduces the level of integration for both modern and traditional workers.

Utility Comparison

Worker type	Concentration vs. Partial Segregation $\sigma = 1.5$			$\sigma = 2$		
	Concentration	Segregation	$\Delta\%$	Concentration	Segregation	$\Delta\%$
Cultural	0.107	0.115	7.15	0.083	0.095	14.85
Modern	0.804	0.854	6.26	0.830	0.943	13.57
Traditional	0.696	0.740	6.26	0.747	0.849	13.57

$H_1 = 1$, $\epsilon = .21$

Worker type	Concentration vs. Partial Integration (Traditional Workers) $\sigma = 1.5$			$\sigma = 2$		
	Concentration	Integration	$\Delta\%$	Concentration	Integration	$\Delta\%$
Cultural	0.113	0.118	4.67	0.088	0.098	11.98
Modern	0.849	0.888	4.67	0.877	0.982	11.98
Traditional	0.736	0.770	4.67	0.789	0.884	11.98

$H_1 = 1.2$, $\epsilon = .21$

Worker type	Concentration vs. Partial Integration (Modern Workers) $\sigma = 1.5$			$\sigma = 2$		
	Concentration	Integration	$\Delta\%$	Concentration	Integration	$\Delta\%$
Cultural	0.109	0.121	11.40	0.088	0.094	7.33
Modern	0.817	0.816	-0.11	0.877	0.915	4.34
Traditional	0.708	0.724	2.25	0.789	0.874	10.65

$H_1 = 1.2$, $\epsilon = .10$

Table 3: Utility comparison moving from concentrated to interior equilibrium
Note: $H_2 = 1$, $\beta = 0.5$, $\delta = .1$, $\gamma = .3$

Table 3 provides the results of the numerical calculations for utility under different equilibrium population distributions. With respect to both the partially segregated and partially integrated equilibrium, concentration is inferior. Furthermore, the utility differential is increasing in σ. Recall that a condition for segregation is that the supply of housing be similar in each region. It follows that concentration generates higher rents in region 1 and workers achieve greater utility by dividing themselves among both regions. This benefit increases with σ, as the value to workers of having access to all of the available varieties diminishes.

In the case of partial integration of modern workers, ϵ is reduced to 0.10 to ensure that condition *ii*) in (36) holds. The results differ from the other two cases. First, for a low value of σ, modern workers would prefer to concentrate in a single region rather than be dispersed. Although modern workers in region 2 receive higher wages than when concentrated, they pay higher rents due to living in a city with a lower supply of housing and have access to a smaller variety of cultural goods than when concentrated. Second, the benefit from integration for cultural workers decreases with σ, which differs from the results in the other two cases. This is due to two factors. First, a decline in σ reduces the market power of cultural producers. Second, it reduces the wages of modern workers which cultural producers rely on for income.

4 Extension: Intensive Margin Choice for Cultural Producers

This section extends the model above by introducing a choice variable for cultural producers to choose the share of their time they devote to cultural production with any remaining time devoted to production of the traditional good. Define θ_i as the share of time that a cultural producer devotes to producing the cultural good in region i and receiving the wage w_{ci} with $1 - \theta_i$ the share spent in the traditional sector and receiving $w_t = 1$. Therefore, a cultural producer's income is given by $\theta_i w_{ci} + (1 - \theta_i)$. Additionally, we make the following assumption,

Assumption 4: $\frac{2\beta}{1-\beta} > 1$.

Assumption 4 is consistent with Assumption 2 above, which ensures that when modern workers concentrate in a single region, they receive a higher wage than traditional workers. These are the only significant changes to the model above. Consider the cultural producers utility maximization problem. The Lagrangian is given by

$$\mathcal{L}_{ci} = T_{ci}^{\alpha} M_{ji}^{\beta} H_{ji}^{\gamma} C_{ji}^{\delta} + \mu_{ci} \left(\theta_i w_{ci} + (1 - \theta_i) - (T_{ci} + p_m M_{ci} + r_i H_{ci} + p_{ci} C_i) \right), \quad (38)$$

where μ_{ci} is the multiplier on the constraint. The first-order condition with respect to θ_i is given by

$$\mu_i (w_{ci} - 1) = 0. \quad (39)$$

Eq.(39) says that when the wage in cultural production is greater than (less than) the wage in the traditional sector the constraint is non-binding, $\mu_i = 0$, and all labor time is devoted to cultural (traditional) production. If the constraint is binding, it must be the case that the unit wage in the cultural sector is equal to that of the traditional sector. We can rule out the case of the cultural sector wage being below that of the traditional sector given that in the absence of workers in the cultural sector, from Cobb-Douglas utility, there would always be sufficient demand to ensure $w_{ci} \geq 1$ and some positive level of the cultural good is produced. The remainder of this section focuses on the case where $\theta_i \in (0, 1]$.

When $\theta_i \in (0, 1)$, $w_{ci} = 1$, therefore the regional prices can be rewritten as

$$p_m = \frac{2\beta}{1-\beta} \frac{1}{\lambda_{m1} A_{m1} + \lambda_{m2} A_{m2}}, \tag{40}$$

$$r_i = \gamma \left(\frac{2\beta}{1-\beta} \frac{\lambda_{m_i} A_{mi}}{\lambda_{m1} A_{m1} + \lambda_{m2} A_{m2}} + \lambda_{ti} + \lambda_{ci} \right) \frac{1}{H_i}. \tag{41}$$

From the market clearing condition for cultural goods we have

$$\theta_i = \frac{\delta}{\sigma} \left(\frac{2\beta}{1-\beta} \frac{\lambda_{m_i} A_{mi}}{\lambda_{m1} A_{m1} + \lambda_{m2} A_{m2}} + \lambda_{ti} + \lambda_{ci} \right) \frac{1}{\lambda_{ci}} \tag{42}$$

The effect of λ_{m1} and λ_{t1} on p_m and r_i are consistent with section 2.3 above. However, notice that rents now include the size of the artists labor force. Additionally, note that θ_i is decreasing in the supply of cultural producers. The greater the local number of artists, the more competitive the market, which reduces the amount of time artists can spend on cultural production.

4.1 Concentration or Segregation: An Artists Colony with Labor Choice

The conditions that ensured that integration was not a viable equilibrium in the preceding model continues to hold in the case where cultural producers can enter the traditional labor sector. We now consider how labor choice affects the stability of a concentrated equilibrium. As in above, suppose all workers choose to concentrate in region 1. Further, consider whether cultural workers would choose to leave the region. If cultural workers are the sole inhabits of region 2, rents are given by $r_2 = \gamma \frac{(1-\lambda_{c1})}{H_2}$ and the labor share devoted to cultural production is constant with $\theta_2 = \frac{\delta}{\sigma}$. The condition for a stable, concentrated equilibrium can be shown to be

$$\lim_{\lambda_{c1}\to 1} \frac{H_1^\gamma}{(\frac{2\beta}{1-\beta}+1+\lambda_{c1})^{\frac{\delta+\gamma(1-\sigma)}{1-\sigma}}} - \frac{H_2^\gamma}{(\frac{\delta}{\sigma})^{\frac{\delta}{1-\sigma}}(1-\lambda_{c1})^{\frac{\delta+\gamma(1-\sigma)}{1-\sigma}}} > 0 \tag{43}$$

When $1+\delta < \sigma < 1+\frac{\delta}{\gamma}$, the concentrated equilibrium is stable. However, when $\sigma > 1+\frac{\delta}{\gamma}$, the condition in (41) fails to hold. In this case, there is an interior equilibrium in which a share of the cultural producers are segregated in region 2. To avoid confusion with the case of partial segregation, the segregated equilibrium is referred to as the artists colony. Solving for λ_{c1} yields,

$$\lambda_{c1}^{AC} = \frac{2(\frac{\delta}{\sigma})^\omega H_2^\psi}{(\frac{\delta}{\sigma})^\omega H_1^\psi + H_2^\psi}\left(\left(\frac{H_1}{H_2}\right)^\psi - \left(\frac{\sigma}{\delta}\right)^\omega \frac{1+\beta}{1-\beta}\right), \tag{44}$$

$$\theta_1^{AC} = 2\left(\frac{\delta}{\sigma}\right)\frac{1}{1-\beta}\left(\frac{\left(\frac{H_1}{H_2}\right)^\psi}{\left(\frac{H_1}{H_2}\right)^\psi - \left(\frac{\sigma}{\delta}\right)^\omega\left(\frac{1+\beta}{1-\beta}\right)}\right), \tag{45}$$

$$\theta_2^{AC} = \frac{\delta}{\sigma}, \tag{46}$$

$$\omega = \frac{\delta}{\delta+(1-\sigma)\gamma} < 0,$$

where the superscript AC is a mnemonic for artists colony and ω is a bundle of parameters. For λ_{c1}^{AC} and θ_1^{AC} to be positive requires

$$\left(\frac{H_1}{H_2}\right)^\psi > \left(\frac{\sigma}{\delta}\right)^\omega \left(\frac{1+\beta}{1-\beta}\right). \tag{47}$$

Given that $\omega < 0$, this condition is more likely to hold when housing supply in region 1 is greater than that of region 2 and/or σ is high. This result differs from the case where the labor supply of cultural workers was inelastic, which ensured that the concentrated equilibrium is always stable. Allowing cultural producers access to the traditional sector labor market generates a minimum level of income for the artist's community, of which a portion is spent on cultural goods. However, it is straightforward to verify that $\theta_1 > \theta_2$, so that isolated cultural producers in region 2 spend a larger share of their time in production of the traditional good. For cultural producers in region 1, access to the larger market and thus greater aggregate demand allows artists to devote a larger share of their time in producing cultural goods.

4.2 Partial Segregation of Traditional Workers with Labor Choice

Again, consider the case where $\lambda_{m1} = 1$ and $\lambda_{t1} = 0$. The interior equilibrium condition for cultural producers is given by

$$\dot{V}_c\Big|_{\substack{\lambda_{m1}=1\\ \lambda_{t1}=0}} = \frac{1}{r_1^{\gamma}(\theta_1\lambda_{c1})^{\frac{\delta}{1-\sigma}}} - \frac{1}{r_2^{\gamma}(\theta_2(1-\lambda_{c1}))^{\frac{\delta}{1-\sigma}}}$$

$$= \frac{H_1^{\gamma}}{\left(\frac{2\beta}{1-\beta}+\lambda_{c1}\right)^{\frac{\delta+\gamma(1-\sigma)}{1-\sigma}}} - \frac{H_2^{\gamma}}{\left(1+(1-\lambda_{c1})\right)^{\frac{\delta+\gamma(1-\sigma)}{1-\sigma}}} = 0. \tag{48}$$

An interior solution is only stable when $\sigma > 1 + \delta/\gamma$. Solving for λ_{c1}, θ_1 and θ_2 yields,

$$\lambda_{c1}^{SC} = \frac{2H_2^{\psi}}{H_1^{\psi}+H_2^{\psi}}\left(\left(\frac{H_1}{H_2}\right)^{\psi} - \frac{\beta}{1-\beta}\right), \tag{49}$$

$$\theta_1^{SC} = \frac{\delta}{\sigma}\frac{H_1^{\psi}}{(1-\beta)H_1^{\psi}-\beta H_2^{\psi}}, \tag{50}$$

$$\theta_2^{SC} = \frac{\delta}{\sigma}\frac{2H_2^{\psi}}{(1+\beta)H_2^{\psi}-(1-\beta)H_1^{\psi}} \tag{51}$$

$$\psi \equiv \frac{\gamma(\sigma-1)}{\gamma(\sigma-1)-\delta}. \tag{52}$$

where SC is used as a mnemonic for segregation with labor choice and ψ is defined as before. Note that for $\lambda_{c1}^{SC} > 0$, $\theta_1^{SC} > 0$ and $\theta_2^{SC} > 0$ requires

$$\frac{1+\beta}{1-\beta} > \left(\frac{H_1}{H_2}\right)^{\psi} > \frac{\beta}{1-\beta}. \tag{53}$$

Notice that θ_i^{AC} is declining in σ and the local supply of housing, while increasing in the supply of housing in the opposite region. Intuitively, when housing supply increases in the opposite region, rents fall there, leading to migration toward that region. In order to keep the cultural and traditional wages equal, cultural workers in the local region offset the decline in the number of artists by increasing the share of time devoted to cultural production. Given that the wage of the cultural and traditional workers are the same it follows that $\dot{V}_t = 0$ and $\dot{V}_m > 0$ ensuring that the equilibrium is stable. Note that

$$\theta_1^{SC} > \theta_2^{SC} \iff \beta > \frac{1 + \left(\frac{H_1}{H_2}\right)^{\psi}}{3 + \left(\frac{H_1}{H_2}\right)^{\psi} + 2\left(\frac{H_2}{H_1}\right)^{\psi}} \tag{54}$$

This simply says that if the expenditure share on modern goods, and therefore the wage and purchasing power of modern workers, is sufficiently high, artists in region 1 spend a greater share of their time on cultural production. If each region has the same supply of housing, the right-hand side of (54) reduces to $\beta > 1/3$ which by Assumption 4 always holds. However, the right-hand side of the second inequality in (54) is increasing in H_1, requiring a larger value of β in order for the condition to hold. Intuitively, a greater supply of housing leads to a larger number of artists. Given the larger supply of workers, cultural producers then reduce the number of hours in order to maintain the equilibrium between the wages in both the cultural and traditional sectors.

4.3 Partial Integration with Labor Choice

Note that when labor choice is introduced for cultural workers, $\dot{V}_c = \dot{V}_t$. This reduces the system of equation by 1, with three equations to solve for 4 variables, ($\lambda_{c1}, \lambda_{t1}, \theta_1$ and θ_2, respectively), leaving the system underdetermined. However, we can consider the case where artists in one region only work in cultural production, while in the other region artists split their time between the cultural and traditional sectors. Consider the case that $\theta_1 = 1$, $\theta_2 \in (0,1)$ and $\lambda_{m1} = 1$. For $\dot{V}_c = 0$ and $\dot{V}_t = 0$, implies that $w_{c1} = 1$.

Solving for the remaining variables yields

$$\lambda_{c1}^{IC} = \frac{2\delta}{\sigma(1-\beta)} \frac{H_1^{\psi}}{H_1^{\psi} + H_2^{\psi}}, \tag{55}$$

$$\lambda_{t1}^{IC} = 2\frac{(\sigma(1-\beta)-\delta)H_2^{\psi}}{\sigma(1-\beta)(H_1^{\psi}+H_2^{\psi})}\left(\left(\frac{H_1}{H_2}\right)^{\psi} - \frac{(\sigma\beta)}{(\sigma(1-\beta)-\delta)}\right), \tag{56}$$

$$\theta_2^{IC} = \frac{2\delta H_2^{\psi}}{\sigma(1-\beta)(H_1^{\psi}+H_2^{\psi}) - 2\delta H_1^{\psi}} \tag{57}$$

where IC denotes integration with labor choice. There is a similar symmetric equilibrium with $\theta_1 \in (0,1)$ and $\theta_2 = 1$. To ensure $\lambda_{t1} > 0$ requires

$$\left(\frac{H_1}{H_2}\right)^{\psi} > \frac{(\sigma\beta)}{(\sigma(1-\beta)-\delta)}, \tag{58}$$

$$\sigma(1-\beta) > 2\delta. \tag{59}$$

The first condition is equivalent to the case for partial integration with inelastic labor supply in (32), while the second condition ensures that there is a positive share of cultural workers in region 2. Note that when $\lambda_{m1} = 1$ and $\theta_1 = 1$, if $(\frac{H_1}{H_2})^{\psi} > \frac{\sigma\beta}{\sigma(1-\beta)-2\delta}$, then $\lambda_{t1} > \lambda_{c1}$. This is in contrast to the results without labor choice, where the larger share of traditional workers settle in the region without modern workers. The intuition is that the supply of the cultural good is dependent on both the number of cultural workers in each city and the number of hours each worker devotes to cultural production. In order for the wage of cultural workers to equalize with those of traditional workers, the supply of cultural output in each region must be adjusted. When workers in one city devote all of their time to cultural production, only a small portion of the labor force can reside there. In contrast, the other city has a larger share of the population but the workers devote a relatively small amount of time to cultural production. It can be verified that when $\sigma(1-\beta) - 2\delta > 0$, then $\lambda_{c1}^{IC} > \theta_2^{IC}\lambda_{c2}^{IC}$. That is, the supply of cultural goods is greater in the region where artists don't divide their time between industries. In this case, traditional workers in region 1 are compensated for higher rents with a greater supply of cultural goods. Additionally, it is worth noting that both partial segregation and partial integration generate the same level of utility for all workers, even though the population distributions vary.

It its shown in Appendix A.2 that there is not a stable partially integrated equilibrium with modern workers. The logic is similar to that of section 2.5.1. In order for traditional workers to be concentrated in one region and cultural workers to be divided, the price index in each city must be equal. Modern workers are always motivated to move away from an even distribution. We now turn to a numerical evaluation of the results.

5 Utility Comparison with Labor Choice

	$\sigma = 1.5$	$\sigma = 2$
Artists Colony		
λ_{c1}^{AC}	0.99	0.39
θ_1^{AC}	0.27	0.44
θ_2^{AC}	0.07	0.05
λ_{t1}^{AC}	1.00	1.00
λ_{m1}^{AC}	1.00	1.00
Partial Segregation		
λ_{c1}^{SC}	0.41	0.21
θ_1^{SC}	0.39	0.53
θ_2^{SC}	0.18	0.11
λ_{t1}^{SC}	0.00	0.00
λ_{m1}^{SC}	1.00	1.00
Partial Integration (Traditional)		
λ_{c1}^{IC}	0.16	0.11
θ_1^{IC}	1.00	1.00
θ_2^{IC}	0.13	0.10
λ_{t1}^{IC}	0.25	0.10
λ_{m1}^{IC}	1.00	1.00

Table 4: Population Shares and Cultural Producers Time Shares.
Note: $H_1 = 1.2$, $H_2 = 1$, $\epsilon = .21$, $\beta = 0.5$, $\delta = .1$, $\gamma = .3$

As in Section 3, we now consider which equilibrium configuration is preferred by workers. Given that the equilibrium requires fewer restrictions on the parameters than the preceding model, we can compare each equilibrium directly. This is due to the fact that changes in the population shares between cities lead to a shift in θ_i, allowing for additional stable configurations of the population for a given set of parameters. The parameters are calibrated using the same set of data as in Table 2.

Table 4 gives the population distributions and the employment time shares of cultural producers. Under the artists colony configuration, 1 % of cultural producers are located in region 2 when σ is low, however, when σ is higher over 60 % locate in region 2. Additionally, as σ increases, the share of time that cultural produces in region 1 devote to artistic production rises from just above a quarter to nearly a half; while for artists in region 2, the share of time devoted to artistic production falls with σ. Under partial segregation, when $\sigma = 1.5$, cultural producers are fairly evenly split between both regions; however, the share in region 1 that are located with modern workers, that receive a wage of twice that of traditional workers, spend nearly double the time on cultural production than those located in region 2. However, as σ increases to 2, just one fifth of cultural producers are in region 1, and they spend half of their time on cultural production. Finally, under partial integration of traditional workers, when artists in region 1 devote all of their time to cultural production, less than one fifth of all cultural workers locate in region 1 and this falls to one tenth as σ increases.

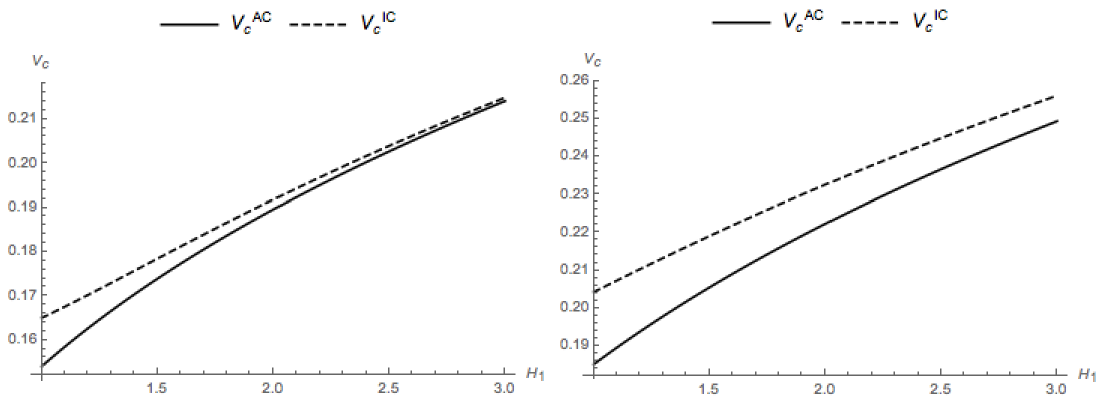
Utility Comparison

Worker Type	Artists Colony vs. Partial Segregation $\sigma = 1.5$			$\sigma = 2$		
	Artists Colony	Segregation	%Δ	Artists Colony	Segregation	%Δ
Cultural	0.163	0.170	4.64	0.194	0.210	8.52
Traditional	0.325	0.340	4.64	0.388	0.421	8.52
Modern	0.163	0.170	4.64	0.194	0.210	8.52

	Artists Colony vs. Partial Integration					
	Artists Colony	Integration	%Δ	Artists Colony	Integration	%Δ
Cultural	0.163	0.170	4.64	0.194	0.210	8.52
Traditional	0.325	0.340	4.64	0.388	0.421	8.52
Modern	0.163	0.170	4.64	0.194	0.210	8.52

Table 5: Utility comparison moving from concentrated to interior equilibrium
Note: $H_1 = 1.2$, $\epsilon = .21$, $H_2 = 1$, $\beta = 0.5$, $\delta = .1$, $\gamma = .3$

Table 5 provides the utility comparison for the different equilibrium configurations. Note that, as mentioned above, both partial segregation and partial integration provide the same level of utility. Additionally, those cases are always preferred to the case of the artists colony. Additionally, the difference is increasing in σ. Notice when labor choice for cultural producers is added to the model there is a significant empirical improvement, with the utility of artists (and traditional workers) half that of modern workers. When labor supply was inelastic in the previous model, cultural worker's utility was roughly one eighth that of modern workers. Finally, while segregation (or integration) is preferred to the artists colony, the difference is declining in H_1 and the two converge in the limit as the supply of housing in region 1 becomes infinite. However, the convergence is slower as σ increases. This is shown in Figure 2, where in panel (a) when $\sigma = 1.5$, as H_1 increases to 3, V_c^{IC} and V_c^{AC} are roughly equivalent. However for a higher value of σ there is a larger gap between the utility levels.



(a) $\sigma = 1.5$ (b) $\sigma = 2$

Figure 2: Utility difference between integration and artists colony

6 Conclusion and Future Research

This paper has developed a spatial model with heterogeneous agents in order to explore the tradeoffs between the market size in aggregate demand and market crowding effects in housing costs that cultural producers face in cities that foster industries with agglomeration economies. We first consider the case where all types of workers are interregionally mobile but intersectorally fixed. The conditions, for the concentration, segregation and integration of workers were analyzed. In particular, it was shown that while concentration is always a stable equilibrium, partial segregation is only stable when agglomeration economies are sufficiently high and each region has a similar supply of housing. Partially integrated equilibria are stable if the elasticity of substitution between varieties of cultural goods and/or the expenditure share on housing is sufficiently high and one region has a significantly greater supply of housing. Additionally, when calibrated, partially segregated and partially integrated equilibria generate higher utility than concentration.

The model was extended to allow for an intensive margin choice by cultural workers on the supply of labor they choose to provide towards cultural production. Notably, concentration is only stable when the elasticity of substitution between cultural goods is sufficiently low. When this fails to hold, there is a stable equilibrium where a portion of cultural workers segregate themselves into the opposite region. Furthermore, the difference between utility levels under each equilibrium configuration is increasing in the elasticity of substitution and decreasing in relative housing supply.

There are two additions that we think would enrich the model. The first is to allow cultural producers to "moonlight" in the modern sector. In practice, there are a number of industries that provide lucrative contract work for artists such as advertising, the film industry, and fashion. Additionally, such work is often more complementary to an artist's skill set. Therefore, it would be useful to consider how artists would choose to divide their time when the working conditions are not simply a tradeoff between the wages in different industries.

The second addition is to introduce a public sector in order to consider how the spatial organization of households leads to changes in the quality of the housing supply. In the model presented above, housing quality and quantity are fixed. However, the quality of housing and the level of public services in a community are determined by its income level. Therefore, by introducing a public sector that attempts to maximize the local utility level through the provision of public services, given the income constraint of the community, we could more fully explore issues of blight and gentrification associated with the migration patterns of cultural producers.

References

[1] Alper, N. O., Wassall, G.H., Jeffri, J., Greenblat, R., Kay, A.O., Butcher, S.G.W., Chartrand, H.H. 1996. Artists in the Work Force:Employment and Earnings 1970 to 1990. NEA Research Division Report No. 37, Washington, DC: National Endowment for the Arts.

[2] Alper, N. O., Wassall, G.H., 2000. More Than Once in a Blue Moon: Multiple Jobholdings by American Artists, National Endowment for the Arts, Santa Ana, CA: Seven Locks Press.

[3] Alper, N.O., Wassal, G.H. 2006. Artists' careers and their labor markets. Reprinted in Ginsburgh and Throsby, eds. *Handbook of the Economics of Arts and Culture*. New York: Elsevier, 813-864.

[4] Bureau of Labor Statistics, US Department of Labor. 2016. Consumer Expenditures-2015. (Press Release) Retrieved from: https://www.bls.gov/news.release/pdf/cesan.pdf

[5] Bureau of Labor Statistics, U.S. Department of Labor. 2015. Consumer spending on entertainment by household income in 2013. *The Economics Daily*. Retrieved from: https://www.bls.gov/opub/ted/2015/consumer-spending-on-entertainment-by-household-income-in-2013.htm (visited May 11, 2017).

[6] Ciccone, A., Hall, R.A. 1996. Productivity and the density of economic activity, *American Economic Review*, 86: 54-70.

[7] Cowen, T. Tabarrok, A. 2000. An economic theory of avant-garde and popular art, or high and low culture. *Southern Economic Journal*. 67: 232-253.

[8] Ewing, H., Grady, R., directors. 2012. Detropia. Loki Films.

[9] Forslid, R., Ottaviano, G.I.P. 2003. An analytically solvable core-periphery model. *Journal of Economic Geography*. 3: 229-240.

[10] Fujita, M., Krugman, P., Venables, A. 1999. *The Spatial Economy*. Cambridge: MIT Press. bibitemrolingstone Geist, W. "Woody Allen: The Rolling Stone Interview". 9th April, 1987. Retrieved from: http://www.rollingstone.com/movies/features/the-rolling-stone-interview-woody-allen-19870409 on May 11, 2017.

[11] Helpman, E. 1998. The size of regions. Reprinted in Pines, Sadka and Zilcha, eds, *Topics in Public Economics*. Cambridge: Cambridge University Press, 33-54.

[12] Krugman, P. 1991. Increasing returns and economic geography, *Journal of Political Economy*, 99: 483-499.

[13] Murata, Y. Thisse, J-F. 2005. A simple model of economic geography à la Helpman-Tabuchi. *Journal of Urban Economics*. 58:137-155.

[14] Pflüger, M., Süedekum, J. 2007. Integration, agglomeration and welfare. *Journal of Urban Economics* 63, 544-566.

[15] Pflüger, M., Tabuchi, T. 2010. The size of regions with land use for production. *Regional Science and Urban Economics* 40, 481-489.

[16] Mancino, L., Richards, T.J. 2011. Demand for food-away-from-home: a multiple discrete/continuous extreme value model. *European Review of Agricultural Economics* 41,111-133.

[17] Tabuchi, T. 1998. Urban agglomeration and dispersion: a synthesis of Alonso and Krugman. *Journal of Urban Economics* 44, 333-351.

[18] Tabuchi, T., Zeng, D. 2004. Stability of spatial equilibrium. *Journal of Regional Science* 44, 641-660.

[19] Throsby, D. 1992. Artists as workers. Reprinted in Towse and Khakee, eds. *Cultural Economics*. New York: Springer-Verlag, 201-208.

[20] U.S. Department of Agriculture, Economic Research Service. (2016) Table 7—Food expenditures by families and individuals as a share of disposable personal income. Retrieved from:https://www.ers.usda.gov/data-products/food-expenditures.aspx

A Appendix

A.1 Equilibrium Conditions for Partial Integration of Modern Workers

This appendix derives the conditions for an equilibrium with in which cultural and modern workers are divided between regions while traditional workers concentrate in region 1. Concentration of traditional workers implies that $P_2 \geq P_1$. Dispersion of modern workers implies that

$$\frac{(1+\lambda_{m1})^{\epsilon}}{(1+(1-\lambda_{m1}))^{\epsilon}} = \frac{P_1}{P_2} \tag{A.1}$$

Therefore, in order for $P_2 \geq P_1$, $\lambda_{m1} \leq 1/2$. From Cobb-Douglas utility, the dispersion of modern workers implies the dispersion of cultural workers. Therefore setting $\dot{V}_c = 0$ we can solve for the number of cultural workers as a function of the number of modern workers

$$\lambda_{c1} = \frac{\left(\left(\frac{\sigma\beta}{\sigma(1-\beta)-\delta}\frac{\lambda_{m1}A_{m1}}{\lambda_{m1}A_{m1}+\lambda_{m2}A_{m2}}+1\right)^{1-\gamma}H_1^{\gamma}\right)^{\frac{1-\sigma}{1-\sigma+\delta}}}{\left(\left(\frac{\sigma\beta}{\sigma(1-\beta)-\delta}\frac{\lambda_{m1}A_{m1}}{\lambda_{m1}A_{m1}+\lambda_{m2}A_{m2}}+1\right)^{1-\gamma}H_1^{\gamma}\right)^{\frac{1-\sigma}{1-\sigma+\delta}}+\left(\left(\frac{\sigma\beta}{\sigma(1-\beta)-\delta}\frac{\lambda_{m2}A_{m2}}{\lambda_{m1}A_{m1}+\lambda_{m2}A_{m2}}\right)^{1-\gamma}H_2^{\gamma}\right)^{\frac{1-\sigma}{1-\sigma+\delta}}} \tag{A.2}$$

Plugging this into $\dot{V}_m$ gives the utility gap solely as a function of λ_{m1}. The migration equation is then determined by

$$sign\,\dot{V}_m = sign\left(\frac{(1+\lambda_{m1})^{\epsilon}H_1^{\frac{\gamma(1-\sigma)}{1-\sigma+\delta}}}{\left(\frac{\sigma\beta}{\sigma(1-\beta)-\delta}\frac{\lambda_{m1}A_{m1}}{\lambda_{m1}A_{m1}+\lambda_{m2}A_{m2}}+1\right)^{\frac{\gamma(1-\sigma)+\delta}{1-\sigma+\delta}}}\right)-\left(\frac{(1+\lambda_{m2}))^{\epsilon}H_2^{\frac{\gamma(1-\sigma)}{1-\sigma+\delta}}}{\left(\frac{\sigma\beta}{\sigma(1-\beta)-\delta}\frac{\lambda_{m2}A_{m2}}{\lambda_{m1}A_{m1}+\lambda_{m2}A_{m2}}\right)^{\frac{\gamma(1-\sigma)+\delta}{1-\sigma+\delta}}}\right) \tag{A.3}$$

Using the fact that $\lambda_{m2} = 1-\lambda_{m1}$, it is clear that concentration in region 1 is not an equilibrium provided that the exponent on the denominators, $(\gamma(1-\sigma)+\delta)/(1-\sigma+\delta)$ is positive. This holds if $\sigma > 1+\delta/\gamma$, which provides condition *i*. If condition *i* holds it follows that $\dot{V}_m|_{\lambda_{m1}=1} < 0$. To ensure that that en equilibrium exists, it must be that case that $\dot{V}_m|_{\lambda_{m1}=0} > 0$. Condition *ii* ensures that this holds. Finally, if an equilibrium exists it needs to be shown that at the equilibrium point $\lambda_{m1} < 1/2$. This can be verified by evaluating $\dot{V}_m$ at $\lambda_{m1} = 1/2$ and ensuring that the utility differential is negative at that point. This is the condition that is given in *iii*.

A.2 Instability of Partial Integration of Modern Workers with Labor Choice

An equilibrium of the partial integration of modern workers requires $\dot{V}_t \geq 0$ and $\dot{V}_c = 0$ which implies $P_1 = P_2$, $\lambda_{m1}^{IM} = 1/2$ and

$$\lambda_{c1}^{IM} = \frac{1+\beta}{1-\beta}\frac{H_1^{\psi} - H_2^{\psi}}{H_1^{\psi} + H_2^{\psi}}$$

where IM denotes partial integration of modern workers. Note that in equilibrium $\dot{V}_t = \dot{V}_c$. If the price indices are equal in both regions, modern workers would have an incentive to move toward one region where they would receive a higher wage as $w_{mi}|_{\lambda_{mi}\in(1/2,1)} > w_{mi}|_{\lambda_{mi}=1/2}$. If they moved toward region 1, this would raise region 1 rents so that $P_1 > P_2$, which would break the equilibrium for traditional workers. If modern workers moved toward region 2, this would raise the price index in region 2 and ensure that the equilibrium holds for traditional workers with $\dot{V}_t > 0$. However since $\dot{V}_t = \dot{V}_c$, this would push all cultural workers to region 1.